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# Nonequilibrium thermodynamics of circulation regimes in optically-thin, dry atmospheres

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## Abstract

An extensive analysis of an optically-thin, dry atmosphere at different values of the thermal Rossby number  $\mathcal{Ro}$  and of the Taylor number  $\mathcal{F}_f$  is performed with a general circulation model by varying the rotation rate  $\Omega$  and the surface drag  $\tau$  in a wide parametric range. By using nonequilibrium thermodynamics diagnostics such as material entropy production, efficiency, meridional heat transport and kinetic energy dissipation we characterize in a new way the different circulation regimes. Baroclinic circulations feature high mechanical dissipation, meridional heat transport, material entropy production and are fairly efficient in converting heat into mechanical work. The thermal dissipation associated with the sensible heat flux is found to depend mainly on the surface properties, almost independent from the rotation rate and very low for quasi-barotropic circulations and regimes approaching equatorial super-rotation. Slowly rotating, axisymmetric circulations have the highest meridional heat transport. At high rotation rates and intermediate-high drag, atmospheric circulations are zonostrophic with very low mechanical dissipation, meridional heat transport and efficiency. When  $\tau$  is interpreted as a tunable parameter associated with the turbulent boundary layer transfer of momentum and sensible heat, our results confirm the possibility of using the Maximum Entropy Production Principle as a tuning guideline in the range of values of  $\Omega$ . This study suggests the effectiveness of using fundamental nonequilibrium thermodynamics for investigating the properties of planetary atmospheres and extends our knowledge of the thermodynamics of the atmospheric circulation regimes.

*Keywords:* Circulation regimes, nonequilibrium thermodynamics,  
terrestrial planetary atmospheres, baroclinic instability, entropy production  
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## 1. Introduction

In the last two decades, more than 700 planets outside the solar system (exoplanets) have been discovered (Udry and Santos, 2007), and the Kepler Space Telescope has recently located over 2,000 exoplanet candidates (Borucki et al., 2011). The study of exoplanets and their climates is in its early stage and it is quickly developing (Seager and Deming, 2010). Observational data are still poor and difficult to obtain, particularly for those planets – super-Earths (Charbonneau et al., 2009) – that might be capable of sustaining liquid water and thus potentially suitable for life. Nevertheless, the discovery of exoplanets is extending the scope of planetary sciences towards the study of the so-called “exoclimates” (Heng, 2012; Burrows et al., 1997; Heng et al., 2011a; Showman et al., 2009; Joshi, 2003; Merlis and Schneider, 2010; Lewis et al., 2010; Pierrehumbert, 2010; Thrastarson and Cho, 2011; Rauscher and Menou, 2012; Dobbs-Dixon et al., 2012). Exoplanets and their atmospheres are in general capable of supporting a broad set of circulation regimes since they are characterized by a range of physical (atmospheric composition, rotation rate, dimension, surface) and orbital (obliquity, eccentricity, distance from the parental star, spectral type of the parental star, presence or not of phase locking) parameters even wider than that of Solar System planets (Williams and Pollard, 2002). Planetary science aims at predicting and classifying in a concise but comprehensive way exoclimates once the main orbital and physical parameters are known.

Recently Read (2011) noted that the large variety of circulation regimes may be better understood by adopting the fluid-dynamical method of similarity, i.e. by defining a set of dimensionless numbers that fully characterise the planetary circulations. Two climate states that share the same set of dimensionless numbers are dynamically equivalent and so the statistical properties of one can be mapped onto those of the other. Obviously the set of parameters is fairly large, and one of the main objectives of planetary science is to understand what is the minimal number of dimensionless parameters needed to define virtually equivalent circulations (Wang, 2012; Showman et al., 2010). In this study we focus on the impact of two parameters, the rotation rate  $\Omega$

and on the surface turbulent exchange rate  $\tau$ , on the atmospheric circulation of an Earth-like dry atmosphere. The choice of such parameters naturally leads to the definition of two dimensionless numbers, the thermal Rossby number  $\mathcal{Ro}$  and the Taylor (frictional) number  $\mathcal{F}_f$  (Read, 2011).

Over the last three decades, the effect of the planetary rotation on atmospheric circulation has been investigated in some details with the aid of general circulation models (Hunt, 1979; Williams, 1988a,b; Navarra and Boccaletti, 2002; Genio and Suozzo, 1987; Geisler et al., 1983; Read, 2011; Vallis and Farneti, 2009). Variations in the value of  $\Omega$  impacts directly the size of the baroclinic waves and the extent of the Hadley cell, which are the main features of the large-scale Earth atmospheric circulation. The size of the baroclinic disturbances, being proportional to the Rossby deformation radius (Eady, 1949), scales as  $1/\Omega$ . The latitudinal extent of the Hadley cell also scales as  $1/\Omega$  (Held and Hou, 1980). Numerical simulations of slowly rotating Earth-like planets and of Solar System planets like Venus and Titan (Clancy et al., 2007; Hourdin et al., 1995) have shown the presence of one poleward-extended Hadley cell in each hemisphere and the weakening or complete disappearing of the midlatitude baroclinic disturbances. On the other hand, at fast rotation rates the emergence of multiple cells in the meridional circulation and multiple jets in the zonal circulation has been observed both in numerical simulations (Williams, 1988a, 1978) and observations (e.g. Jupiter).

The dynamical effects of the solid lower boundary of terrestrial planets on the atmospheric circulation is also quite important in order to understand planetary circulations and has not been fully addressed yet (Showman et al., 2010). The characteristics of the surface have been recognised as a key factor in shaping Earth’s atmospheric circulation (James, 1994; James and Gray, 1986), although this topic has received less attention than that related to  $\Omega$ . The surface of a terrestrial planet, due to its roughness, affects the turbulent flow within the planetary boundary and thus the exchange of momentum and energy between the surface and the atmosphere (Arya, 1988). It has been shown (James and Gray, 1986; James, 1987; Kleidon et al., 2003) that the reduction of the surface drag leads to strong horizontal barotropic shears in the zonal mean flow. By using a two-level quasi-geostrophic model, James (1987) showed that the growth rate of the most unstable baroclinic modes is reduced considerably by the strong horizontal wind shears. This is related to the general fact that the linearised baroclinic instability equations obey the Squire’s theorem (Kundu and Cohen, 2004). The role of drag has received

71 some attention in the exoplanets context (Rauscher and Menou, 2012) but,  
72 to the authors' knowledge, has not been systematically investigated so far  
73 for rotation rates which are different from the Earth's. In this study we  
74 investigate the combined effect of rotation speed and surface roughness on  
75 the dynamics, linking it to the nonequilibrium thermodynamics of the system.  
76 Thermodynamics provide a way for characterizing concisely a complex  
77 physical system, bringing together comprehensive but minimal physical in-  
78 formation. The atmosphere of a planet is an example of a nonequilibrium  
79 system (Gallavotti, 2006; DeGroot and Mazur, 1984; Kleidon, 2009), and its  
80 general circulation redistributes energy in order to compensate for the ra-  
81 diative differential heating between hot and cold regions. The atmospheric  
82 circulation therefore is fuelled by the conversion of available potential energy  
83 due to large temperature gradients into kinetic energy. The atmosphere, in  
84 other terms, produces mechanical work, acting as a heat engine (Lorenz,  
85 1967; Peixoto et al., 1991; Johnson, 2000; Lucarini, 2009). It seems therefore  
86 natural to adopt nonequilibrium thermodynamics as a general framework for  
87 studying exoclimates. Such an approach has been, for example, applied in  
88 Lucarini et al. (2010) and Boschi et al. (2012) for studying the bistability of  
89 an Earth-like planet. Furthermore, thermodynamical disequilibrium drives  
90 a variety of irreversible processes, from frictional dissipation to chemical re-  
91 actions. The irreversibility of climatic processes is quantified by the mate-  
92 rial entropy production (Goody, 2000; Kleidon and Lorenz, 2005; Kleidon,  
93 2009). The interest in studying climate material entropy production largely  
94 stemmed from the proposal of the maximum entropy production principle  
95 (MEPP) by Paltridge (Paltridge, 1975, 1978, 2001), who suggested that the  
96 climate adjusts in such a way as to maximize the material entropy produc-  
97 tion. In its weak form, the MEPP suggests to use the entropy production  
98 as a target function to be maximized when tuning an empirical or uncertain  
99 parameter of a model (Kleidon et al., 2003; Kunz et al., 2008). Whereas the  
100 theoretical foundations of MEPP are still unclear (Dewar, 2005; Grinstein  
101 and Linsker, 2007; Goody, 2007), such a conjecture has also been proposed  
102 as a way to estimate the meridional heat transport of other planets, such  
103 as Mars and Titan (Lorenz et al., 2001; Jupp and Cox, 2010) and poten-  
104 tially to exoplanets too, and has stimulated the re-examination of climatic  
105 dissipative processes (Peixoto et al., 1991; Goody, 2000; Pauluis and Held,  
106 2002a,b; Kleidon and Lorenz, 2005; Fraedrich and Lunkeit, 2008; Pascale  
107 et al., 2011a).  
108 In this study we perform a large ensemble of numerical simulations with

an Earth-like general circulation model for many different values of  $\Omega$  and  $\tau$  in order to compute the dissipative properties  $\zeta$  (where  $\zeta$  is any dissipative function, e.g. material entropy production) of circulations of dry atmospheres at different thermal Rossby and Taylor numbers,  $\zeta(\mathcal{R}o, \mathcal{F}_f)$ . We relate, for the first time, the properties of  $\zeta(\mathcal{R}o, \mathcal{F}_f)$  to the different circulation regimes and extend our knowledge on the global thermodynamic properties of rotating fluids. We anticipate that particular regimes (e.g. baroclinic, zonostrophic, super-rotation) are effectively characterized in terms of their thermodynamic properties. We conclude with a brief analysis of how effectively the MEPP can be used to infer the optimal value for an uncertain or empirical parameter, in this case exactly the time scale controlling the exchange of momentum and energy between free atmosphere and the surface.

The paper is organized as follows, In Section 2 we will shortly discuss the dimensionless parameters relevant for this study. In Section 3 the model and the experimental setup are presented. The characterization of different dynamical regimes is the subject of Section 4 whereas in Section 5 the thermodynamical properties of the circulation regimes are analysed. In Section 6 the main conclusions are summarized.

## 2. Parametric range of general circulations and dimensionless numbers

The role of the rotation rate in planetary circulations has been first investigated in laboratory experiments with a thermally driven rotating annulus (Hide, 1953, 1969; Hide and Mason, 1975; Read et al., 1998; Read, 2001; Wordsworth et al., 2008; Hide, 2010). The system consists of a fluid confined between coaxial cylinders maintained at two different temperatures and rotating at an angular velocity  $\Omega$ . When the basic parameters  $\Omega$  and  $\Delta T$  (temperature difference between the inner and outer cylinder) are varied, a wide variety of flow patterns is observed. Different dynamical regimes can be identified if results are grouped with respect to two dimensionless parameters, the *thermal Rossby number*:

$$\mathcal{R}o = \frac{g\alpha D\Delta T}{\Omega^2 L^2}, \quad (1)$$

and the *Taylor number*:

$$\mathcal{T}a = \frac{4\Omega^2 L^5}{\nu^2 D}, \quad (2)$$

140 in which  $L$  is the channel width,  $D$  its depth,  $\nu$  the kinematic viscosity  
 141 of the fluid,  $\alpha$  its volumetric expansion coefficient, and  $g$  the gravitational  
 142 acceleration.

143 Read (2011) has extended the definition of the thermal Rossby number  
 144 and of the Taylor number to the case of atmospheric circulations. The anal-  
 145 ogous of the thermal Rossby number is defined as:

$$\mathcal{R}_o = \frac{R\Delta\theta_h}{\Omega^2 a^2}, \quad (3)$$

146 where  $a$  is the planet’s radius,  $R$  the specific gas constant and  $\Delta\theta_h$  the hor-  
 147 izontal (potential) temperature contrast between equator and poles. A dif-  
 148 ference between the definitions in eq. (1) and eq. (3) is that  $\Delta\theta_h$  is not  
 149 fixed externally but rather determined by the circulation itself. In the fol-  
 150 lowing we will take  $\Delta\theta_h = \Delta\theta_{hE}$ , as done for example in Mitchell and Vallis  
 151 (2010), where  $\theta_{hE}$  is the radiative-convective equilibrium potential temper-  
 152 ature, since this is externally determined by the incoming stellar radiative  
 153 energy and thus a more objective quantity to describe the horizontal differ-  
 154 ential driver for the circulation. A Taylor number can be defined analogously  
 155 to the case of the rotating annulus as:

$$\mathcal{F}_f = 4\Omega^2 \tau_f^2 \quad (4)$$

156 in which  $\tau_f$  is the typical timescale for kinetic energy dissipation. We note  
 157 that  $\mathcal{F}_f \propto (\tau_f/\tau_{rot})^2$ , where  $\tau_{rot} = 2\pi/\Omega$ , i.e.  $\mathcal{F}_f$  is proportional to the  
 158 ratio of (the squares of) the typical timescales associated with turbulent  
 159 dissipation of kinetic energy and rotation. For planets with a solid core,  $\tau_f$  is  
 160 the surface drag timescale and is in general determined by the characteristics  
 161 of the surface. The use of (3) and (4) has been proved to be very useful in  
 162 classifying atmospheric circulation (Wang, 2012).

### 163 3. Model and experimental setup

#### 164 3.1. The Planet Simulator

165 Numerical simulations have been performed with the Planet Simulator  
 166 (PlaSim), a general circulation model of intermediate complexity (Fraedrich  
 167 et al., 2005). The model is freely available at [www.mi.uni-hamburg.de/plasim](http://www.mi.uni-hamburg.de/plasim).  
 168 PlaSim is a fast running model and it is therefore suitable for large-ensemble  
 169 numerical experiments. Moreover, a full set of thermodynamic diagnostics



170 is available, thus making it well suited for this work (Fraedrich and Lunkeit,  
171 2008; Lucarini et al., 2010).

172 The atmospheric dynamic core uses the primitive equations, which are  
173 solved using a spectral transform method (Eliassen et al., 1970; Orszag, 1970).  
174 Interaction between radiation and atmosphere is dealt with using simple but  
175 realistic longwave (Sasamori, 1968) and shortwave (Lacis and Hansen, 1974)  
176 radiative schemes. In particular the incoming solar flux  $F_{SW}^{toa}$  at the top of  
177 the atmosphere (TOA) is

$$F_{SW}^{toa} = S_0 \cos Z \quad (5)$$

178 where  $S_0$  is the solar constant ( $1365 \text{ W m}^{-2}$ ) and  $Z$  the zenith angle, which  
179 is in general a function on the latitude, time of the year and time of the  
180 day, and it is computed following Berger (1978). All simulations have been  
181 performed with orbital parameter – obliquity, eccentricity, distance from the  
182 Sun, typical of Earth. Other sub-grid scale parametrisations include interac-  
183 tive clouds (Stephens, 1978; Stephens et al., 1982; Slingo and Slingo, 1991),  
184 moist (Kuo, 1965, 1974) and dry convection, large scale precipitation, bound-  
185 ary layer fluxes and vertical and horizontal diffusion (Louis, 1979; Louis et al.,  
186 1981; Laursen and Eliassen, 1989). More information can be found in PlaSim  
187 reference manual, freely available at [www.mi.uni-hamburg.de/Downloads-un.245.0.html](http://www.mi.uni-hamburg.de/Downloads-un.245.0.html).  
188

189 In all simulations the lower boundary is a flat surface with prescribed  
190 albedo and heat capacity (see Table 1). This is implemented with a shallow  
191 energy-conserving slab-ocean model with an areal heat capacity ( $C_{slab} = 10^7$   
192  $\text{J K}^{-1} \text{m}^{-2}$ ) comparable to that chosen in Frierson et al. (2006) and Heng et al.  
193 (2011b). In this way we avoid fixed surface temperature and have a simple  
194 but energetically consistent climate model. The surface temperature evolves  
195 in time according to  $C_{slab} \dot{T}_s = F_{SW}^{surf} + F_{LW}^- = \sigma T_s^4 - F_T$  ( $F_{SW}^{surf}$  net solar  
196 radiation at the surface,  $F_{LW}^-$  downward longwave radiation at the surface,  
197  $F_T$  surface sensible heat flux). We set the depth of the mixed layer to 5 m  
198 in order to have an areal heat capacity ( $C_{slab} = 10^7 \text{ J K}^{-1} \text{m}^{-2}$ ) comparable  
199 to that chosen in Frierson et al. (2006) and Heng et al. (2011b). We have  
200 checked our result at  $C_{slab} = 10^8 \text{ J K}^{-1} \text{m}^{-2}$  too, finding little effects on the  
201 circulations and on the global thermodynamical properties. Simulations are  
202 performed at T42 spectral resolution ( $2.8^\circ \times 2.8^\circ$ ) with ten levels (T42/10LEV  
203 in the following).

204 In this study we consider dry atmospheres. Dry atmospheres are relevant  
205 for planetary (e.g. Mars) and paleoclimatological (e.g. Snowball Earth) stud-

ies and, moreover, allow us to avoid the role of phase transitions associated with condensing substances, simplifying the problem and making neater the connection between dynamics and thermodynamics of the system. Such configuration is obtained by switching off the surface evaporation module and starting from a dry atmospheric condition. Water vapour is consequently not inserted within the atmosphere, which remains dry for all timesteps.

### 3.2. The strength of the turbulent surface exchanges

In order to have a wide and controlled variation in  $\mathcal{F}_f$  (Eq. 4), we simplify the representation of the surface fluxes. In PlaSim the temperature tendency of the first atmospheric layer (of thickness  $dz$ ) due to the turbulent sensible heat flux,  $(\partial T/\partial t)_{shf}$ , is computed as:

$$\left(\frac{\partial T}{\partial t}\right)_{shf} = -\frac{F_T}{\rho c_p dz} = \frac{\gamma_h |\mathbf{u}|}{dz} (T_s - \xi T) = \frac{T_s - \xi T}{\tau_h(\mathbf{x}, t)}, \quad (6)$$

in which  $F_T = \gamma_h |\mathbf{u}| (T_s - \xi T)$  is the surface sensible heat flux,  $\gamma_h = (k/\ln(z/z_0))^2 f(Ri, z_0)$  is the heat transfer coefficients ( $z$  is height from the surface,  $k$  is the von-Karman parameter,  $z_0$  is the surface roughness, and  $f$  is an empirical function dependent on stability (as expressed by the Richardson number  $Ri$ ) and surface roughness),  $\xi$  is the Exner factor (for more details see Louis, 1979; Lunkeit et al., 2010). The parameter  $\tau_h$  has time dimension and in a standard run is a function of space and time,  $\tau_h(x, y, z, t) = dz/(\gamma_h(x, y, t)|\mathbf{u}(x, y, t)|)$  but remains of the same order of magnitude. Since we are interested in variations of orders of magnitude in  $\tau_h$ , we substitute the locally computed  $\tau_h$  with a fixed (in space and time) time scale  $\tau_h$  as:

$$\left(\frac{\partial T}{\partial t}\right)_{shf} = -\frac{\xi T - T_s}{\tau_h}. \quad (7)$$

Similarly to eq. (6), for the wind tendency due to the surface stress,  $(\partial \mathbf{u}/\partial t)_{stress}$ , we have:

$$\left(\frac{\partial \mathbf{u}}{\partial t}\right)_{stress} = -\frac{\mathbf{u}}{\tau_m(\mathbf{x}, t)}. \quad (8)$$

with  $\tau_m(x, y, z, t) = dz/(\gamma_m(x, y, t)|\mathbf{u}(x, y, t)|)$  and the drag coefficient  $\gamma_D$  defined similarly to  $\gamma_h$ . Again we substitute the locally compute  $\tau_m(\mathbf{x}, t)$  with a fixed (in space and time) drag timescale  $\tau_m$  (Rayleigh friction timescale). Generally the drag and heat transfer coefficients  $\gamma_D$  and  $\gamma_h$  – and therefore

the time constants  $\tau_m$  and  $\tau_h$  – have similar magnitude. This is particularly true in the case of neutral flows, for which  $\gamma_D = \gamma_h$  is indeed a very good approximation (Arya, 1988; Louis, 1979). For non-neutral flows,  $\gamma_h$  and  $\gamma_D$  are different but still of the same order of magnitude, as can be seen in Fig. 11.6 of Arya (1988). On the base of this and since in this study we are going to explore a wide parametric range, we assume for the sake of simplicity:

$$\tau_m = \tau_h = \tau. \quad (9)$$

Experiments are performed for  $\Omega^* = \Omega/\Omega_E = 1/10, 1/5, 1/2, 1, 2, 4, 8$ , where  $\Omega_E$  is the Earth rotation rate. For each value of  $\Omega^*$  we run the model with  $\tau = 2700, 3600, 10800, 21600, 43200, 86400, (86400 \times 3), (86400 \times 10), (86400 \times 30), (86400 \times 100), (86400 \times 500)$  seconds, that is from 45 minutes (model timestep for  $\Omega/\Omega_E \leq 1$ ) to 500 days. Simulations with very large  $\tau$  are representative of an atmosphere with no solid lower boundary (James, 1994; Menou and Rauscher, 2009; Heng et al., 2011b).

Let us note that as  $\Omega$  increases, the typical size of the baroclinic disturbances  $L_c$  decreases as (Eady, 1949)

$$L_c = 2.4\pi L_R, \quad (10)$$

with the Rossby deformation radius  $L_R = NH/f$  (James, 1994; Williams, 1988a),  $N$  the buoyancy frequency,  $H$  the height scale and  $f = 2\Omega \sin \varphi$  the Coriolis parameter. For our dry-atmosphere simulations an order-of-magnitude estimate at the midlatitudes for  $\Omega^* = 8$  leads to  $\Delta\theta \approx 110$  K,  $\bar{\theta} \approx 240$  K (see, e.g., Fig.3(h)),  $\Delta z = 9$  km,  $N \approx (g/\bar{\theta}(\Delta\theta/\Delta z))^{1/2} \approx 2 \times 10^{-2}$  s<sup>-1</sup> and therefore to  $L_R \sim 200$  Km. This implies that T42 simulations (spatial resolution about 250 Km) should be able to capture at least the largest eddies at  $\Omega^* = 8$  and more than adequate for  $\Omega^* \leq 4$ .

#### 4. Circulation regimes at different $\mathcal{Ro}$ and $\mathcal{F}_f$

The diagram in Fig. 1(b) shows the dimensionless space  $(\mathcal{F}_f, \mathcal{Ro})$ . The over-plotted bullet points represent numerical experiments performed at  $\Omega^* = 0.1$  (circles, denoted as “slow rotation”),  $\Omega^* = 1$  (squares, “intermediate rotation”) and  $\Omega^* = 8$  (triangles, “fast rotation”) for strong, intermediate and weak drag condition ( $\tau$  equal to 45 minutes, 1 day and 500 days respectively) whose mean meridional and zonal circulations are shown in Fig. 2 and Fig. 3

and delimit the portion of the  $(\mathcal{F}_f, \mathcal{R}o)$  space covered by the numerical simulation performed in this study. We have over-plotted the corresponding values of  $\Omega^*$  (horizontal dot-dashed lines) and  $\tau$  (dotted lines) in order to highlight the connection between the dimensionless numbers and the physical parameters  $\Omega^*$  and  $\tau$ . Note that  $\Omega^*$  and  $\mathcal{R}o$  as well as  $\tau$  and  $\mathcal{F}_f$  point in opposite directions. In order to help to set the stage for the reader to understand the results in the following and make it easier to interpret the montage of figures (3) and (2), we anticipate the main characteristics of the simulated circulations:

1. At high thermal Rossby number ( $\mathcal{R}o \geq 8$ ), the decrease of the surface drag controls the transition from counter- to super-rotating (SR in Fig.1(a)) equatorial flow. Super-rotation is approached for  $\mathcal{F}_f \geq 10^4$ ;
2. At intermediate rotation speed ( $1 \leq \mathcal{R}o \leq 0.01$ ), strong drag ( $\mathcal{F}_f \leq 10$ ) is associated with axisymmetric circulations (AR in Fig. 1(a)). The decrease of  $\tau$  leads to the appearance of the indirect Ferrel cell for  $10 \leq \mathcal{F}_f \leq 10^5$  characterized by baroclinic activity (BC in Fig.1(a)); further decrease of the surface drag ( $\mathcal{F}_f \geq 10^5$ ) leads to the emergence of a barotropic flow (BT in Fig.1(a)) characterised by a large reduction in the vertical shears of the zonal wind and the complete disappearing of the Ferrel cell;
3. For fast rotations ( $\mathcal{R}o \leq 10^{-3}$ ) the increase the of Taylor frictional number ( $\mathcal{F}_f > 10^4$ ) leads to the appearance of a multi-jet, zonostrophic flow (ZN in Fig. 1(a)) for  $\mathcal{F}_f > 10^3$  ( $\tau > 6$  hours)

Boundaries between the different regimes are schematically sketched in Fig. 1(a). In the following we give a detailed description of the different regimes.

#### 4.1. Slow rotation ( $\mathcal{R}o = 8$ )

Fig. 2(a), 2(b), 2(c) and Fig. 3(a), 3(b), 3(c) show the slow rotation rate ( $\mathcal{R}o = 8$ ). Such circulations are dominated by one Hadley cell in each hemisphere which extends northward up to the poles (this regime is denoted AS in the Fig.1(a)). This is a general consequence of the conservation of angular momentum and in agreement with the theory of the Hadley circulation of Held and Hou (1980). The temperature features almost no latitudinal dependence, especially in the middle atmosphere. This is typical of slowly rotating planets (Williams, 1988a; Navarra and Boccaletti, 2002), and is due

297 to the strong Hadley cell circulation. It is interesting to note the effect  
 298 of the surface drag on shaping the Hadley circulation. By comparing Fig.  
 299 2(e) to Fig. 2(b) ( $\mathcal{R}o, 10^{-1} \rightarrow 8$ ;  $\mathcal{F}_f, 10^3 \rightarrow 10$ ) and Fig.2(c) to Fig.2(f)  
 300 ( $\mathcal{R}o, 10^{-1} \rightarrow 8$ ;  $\mathcal{F}_f, 10^7 \rightarrow 10^5$ ) we note a decrease of the counter-rotating  
 301 westward upper-level equatorial jet approaching the beginning of the equa-  
 302 torial super-rotation (for example compare Fig 2(c) to Fig. 13 of Heng and  
 303 Vogt, 2011). Equatorial super-rotation is indeed expected to take place when  
 304  $\mathcal{R}o \gg 1$  (Mitchell and Vallis, 2010). Therefore simulations with  $\Omega^* < 1/10$   
 305 and moderate or high drag are needed in order to obtain fully super-rotating  
 306 atmospheric circulations (as is the case, for example, for Venus to Titan).

#### 307 4.2. Intermediate rotation ( $\mathcal{R}o = 0.08$ )

308 In the medium rotation case ( $\mathcal{R}o = 0.08$ ), we have atmospheric circu-  
 309 lations characterized by strong eastward zonal jets at about  $50 - 60^\circ$  and  
 310 by a thermally direct (Hadley) and indirect (Ferrel) meridional cell (Fig. 2  
 311 (d,e,f) and Fig. 3 (d,e,f)). The general circulation is considerably affected  
 312 by the different surface properties. In particular we note that at large  $\mathcal{F}_f$ ,  
 313 the flow develops strong barotropic horizontal shears, as first discussed by  
 314 James and Gray (1986). Note that, as we are considering a dry optically-thin  
 315 atmosphere, none of the three circulations shown in Fig. (2(d)-2(f)) is close  
 316 to the one we observe on Earth (e.g. Peixoto and Oort (1992)) but rather  
 317 similar to that of Mars (Lewis et al., 2010).

318 The effect of the surface drag is particularly evident in the meridional  
 319 circulation, which is largely modified by the surface properties. A clear  
 320 thermally direct-indirect cell structure emerges in the intermediate cases  
 321  $\mathcal{F}_f \sim 10^2$  ( $\tau \sim 1$  day), with the boundaries of the Hadley cell at about  
 322  $40^\circ$ . The intensity and the extent of the indirect cell is greatly reduced  
 323 in the high drag ( $\mathcal{F}_f \leq 10^{-1}$ ) case, when the baroclinic waves are largely  
 324 suppressed and the flow tend to become axisymmetric. The Ferrel cell is  
 325 instead completely suppressed in the low drag ( $\mathcal{F}_f \geq 10^5$ ) case, where the  
 326 flow becomes barotropic. The large impact of the surface properties on the  
 327 meridional circulation is related to their impact on the baroclinic distur-  
 328 bances (James and Gray, 1986), which normally develop at the edge of the  
 329 thermally direct (Hadley) and indirect (Ferrel) cells. The Ferrel cell is re-  
 330 lated to the presence of eddy momentum convergence, a key ingredient of  
 331 baroclinic disturbances (Holton, 2004), and its disappearance points out the  
 332 suppression or weakening of the midlatitude disturbances. In the presence of  
 333 weak surface drag, zonal winds tend to have high values at the surface which

334 remain fairly constant with height but change sign at the midlatitudes from  
 335 westward to eastward going from the equator to the poles (e.g. Fig. 2(f))  
 336 thus generating a strong horizontal shear. Such strong horizontal shears in-  
 337 hibit the growth of baroclinic waves, as demonstrated in (James, 1987). On  
 338 the other hand, with a surface characterized by a high drag, baroclinicity  
 339 is suppressed too, because the system frictional dissipation is too high and  
 340 kinetic energy is rapidly extracted not giving eddies the chance to grow and  
 341 develop (Kleidon et al., 2003).

342 Let us also note in Fig. 3 the presence of shallow cells embedded close  
 343 to the surface embedded in a larger one. This is a characteristic of optically-  
 344 thin atmospheres of rocky planets in which the solid lower boundary with low  
 345 thermal inertia respond very quickly to diurnal and seasonal solar heating  
 346 (Caballero et al., 2008). Similar features are indeed observed in Mars circu-  
 347 lation (see e.g. figure 2 of Lewis et al., 2010). Such shallow cells disappears  
 348 in fact in the additional runs we have performs at  $C_{slab} = 10^8 \text{ J K}^{-1} \text{ m}^{-2}$  (not  
 349 shown) and have very little effect on the thermodynamic properties we are  
 350 going to discuss in the following sections.

### 351 4.3. Fast rotation ( $Ro = 10^{-3}$ )

352 Finally, in the fast rotation runs ( $Ro = 10^{-3}$ ) we observe multiple jets  
 353 (Fig. 2(g)-(i)) and multiple meridional cells (Fig. 3(g)-(i)) in agreement with  
 354 previous studies (Hunt, 1979; Williams, 1988a) and with the scaling of the  
 355 Rossby deformation radius (eq. 10). The decrease of  $L_R$  with the rotation  
 356 rate makes baroclinic waves less and less efficient in the poleward heat trans-  
 357 porting process and reduction of the meridional temperature contrast. The  
 358 temperature field in fact shows larger contrast in the meridional and vertical  
 359 profile, and the thermal structures tend to be in radiative-convective equi-  
 360 librium. The effect of  $\tau$  is mainly observed in the zonal wind profiles (Fig.  
 361 2 (g,h,i)) and in the meridional stream function (Fig. 3(g,h,i)). Multi-jet,  
 362 zonostrophic flow (Wang, 2012) emerges as the surface drag decreases for  
 363  $\mathcal{F}_f > 10^3$ , as can be seen in Fig. 3(i).

## 364 5. Thermodynamic analysis

### 365 5.1. Thermodynamic diagnostics

366 The general circulation is the result of the conversion of the available  
 367 potential energy generated by radiative differential heating into mechanical  
 368 work (winds), as first shown by Lorenz (1955, 1960, 1967). For an atmosphere

in a statistical steady state, the rate of generation of available potential energy,  $G$ , the rate of conversion into kinetic energy,  $W$ , and the rate of dissipation of kinetic energy through the turbulent cascade (and ultimately via viscous dissipation),  $D$ , have to be equal when averaged over long time periods (e.g. a year or longer),  $\overline{G} = \overline{W} = \overline{D}$  ( $\overline{(\cdot)}$  denotes the time mean). They are therefore equivalent ways of measuring the strength of the Lorenz energy cycle (Lorenz, 1955).

The energy cycle introduced by Lorenz has been set onto a thermodynamic framework through the consideration of the effective Carnot engine describing the ability of the atmosphere to perform work (Johnson, 2000; Adams and Rennó, 2005; Lucarini, 2009). The atmosphere is seen as a heat engine which generates mechanical work at average rate  $\overline{W}$  from the differential heating due to radiative and material (e.g. latent heat release) diabatic processes. If  $\dot{Q}^+$  and  $\dot{Q}^-$  are the local positive and negative diabatic heating rate (i.e.  $\dot{Q}^+ = \dot{Q}$  where  $\dot{Q} > 0$  and  $\dot{Q}^+ = 0$  where  $\dot{Q} < 0$  and similarly for  $\dot{Q}^-$ ) with

$$\Phi^\pm = \int \dot{Q}^\pm \rho dV, \quad (11)$$

we have that  $\overline{\Phi^+} + \overline{\Phi^-} = \overline{W} \geq 0$ . Moreover, one can define an efficiency  $\eta$ :

$$\eta = \frac{\overline{\Phi^+} + \overline{\Phi^-}}{\overline{\Phi^+}} \quad (12)$$

which gives us an indication about the capability of the general circulation of generating kinetic energy given the net heating input  $\Phi^+$ . From Eq. (7) it follows that

$$\overline{W} = \eta \overline{\Phi^+} \quad (13)$$

in full analogy with the definition of efficiency of a heat engine (Fermi, 1956). Such a quantity has been proved to be particularly relevant in marking the climatic shifts between the present day climates and the Snowball Earth (Lucarini et al., 2010; Boschi et al., 2012)

Dissipation, and therefore irreversibility, is ubiquitous in planetary atmospheres and, more generally, in nonequilibrium systems. The kinetic energy of the atmospheric flow is ultimately transferred through a turbulent cascade to smaller scales where it is then dissipated into heat by friction due to viscosity. Thermal dissipation due to sensible heat fluxes between the surface and lower atmosphere is another irreversible process which may take place

399 in planetary atmospheres. Planets whose atmospheres allow phase transi-  
 400 tions of one or more of their chemical substances (e.g. water on Earth or  
 401 methane on Titan) also experience further irreversible processes as evapo-  
 402 ration/condensation and diffusion (Goody, 2000; Pauluis and Held, 2002b).  
 403 Irreversible processes are associated with a positive-defined material entropy  
 404 production (Peixoto et al., 1991; DeGroot and Mazur, 1984; Kondepudi and  
 405 Prigogine, 1998; Fraedrich and Lunkeit, 2008; Kleidon, 2009). General dis-  
 406 cussions about the entropy budget of the climate system and about how to  
 407 estimate it from climate models can be found in Peixoto et al. (1991), Goody  
 408 (2000), Kleidon and Lorenz (2005), Kleidon (2009), Pascale et al. (2011a),  
 409 Pascale et al. (2011b), Lucarini et al. (2011). For a climate with a dry at-  
 410 mosphere the material entropy production is due to two kinds of processes:  
 411 dissipation of kinetic energy and sensible heat fluxes. If  $\epsilon^2$  is the local rate  
 412 of kinetic energy dissipation such that  $D = \int \epsilon^2 \rho dV$ , the entropy production  
 413 associated with it reads:

$$\dot{S}_{kedis} = \int \frac{\epsilon^2}{T} \rho dV. \quad (14)$$

414 In PlaSim the dissipation of kinetic energy is due to: (i) turbulent stresses in  
 415 the surface boundary layer (which accounts for more than 50% of the overall  
 416 dissipation) and, gravity wave drag, implemented as a Rayleigh friction at  
 417 the highest level with a timescale of 50 days, which we define as  $D_{phys}$ .  
 418 Such contribution to the total mechanical dissipation is diagnosed in the  
 419 model as  $1/2 \int \rho dz (\mathbf{v}_a^2 - \mathbf{v}_b^2)$  where  $\mathbf{v}_b$  and  $\mathbf{v}_a$  is the velocity before and  
 420 after the application of the boundary layer scheme and Rayleigh friction;  
 421 (ii) numerical dissipation due to numerical diffusion of momentum (Johnson,  
 422 1997), which we call  $D_{num}$ . More precisely, in PlaSim horizontal diffusion  
 423 is implemented by a 8th order hyperdiffusion term applied to the vertical  
 424 component of the relative vorticity  $\zeta = \mathbf{k} \cdot (\nabla \times \mathbf{v})$  and horizontal wind  
 425 divergence  $\delta = \nabla_h \cdot \mathbf{v}$ ,  $\kappa \nabla^8(\zeta, \delta)$ , where  $\kappa$  is a coefficient of numerical diffusion  
 426 – the prognostic equations for the horizontal velocity are transformed into  
 427 equations for  $\zeta$  and  $\delta$ , for more details on PlaSim dynamical core see Lunkeit  
 428 et al. (2010) –. Although it is hard to interpret  $D_{num}$  as representative of  
 429 small scale dissipative processes (Jablonowski and Williamson, 2011) – the  
 430 hyperdiffusion schemes do not usually match the symmetry requirements of  
 431 the stress tensor needed to ensure the conservation of the angular momentum  
 432 (Becker, 2001) – these contributions are produced by the model and will be  
 433 taken into account in order to be consistent with the model itself (Johnson,



1997; Egger, 1999; Woollings and Thuburn, 2006). The total dissipation of kinetic energy of the model is therefore  $D = D_{phys} + D_{num}$ .

Sensible heat in PlaSim is associated with turbulent surface fluxes  $F_T$  driven by the temperature difference existing between the lowermost part of the atmosphere and the surface and with numerical vertical and horizontal diffusion (of the same kind of that used for momentum) and dry convection. The material entropy production associated with  $F_T$  is:

$$\dot{S}_F = \int F_T \left( \frac{1}{T_a} - \frac{1}{T_S} \right) dA, \quad (15)$$

where  $T_a$  is the temperature of the first atmospheric level (where  $F_T$  is absorbed thus heating it) and  $T_S$  the surface temperature. The material entropy production associated therefore to sensible heat is the sum of the material entropy production due to surface turbulent fluxes,  $\dot{S}_{sens}$  and to the other sources of sensible heat (diffusion and dry convection),  $\dot{S}_{sens}$ , and it reads

$$\dot{S}_{sens} = \dot{S}_F + \dot{S}_{diff}. \quad (16)$$

The total material entropy production of the system is therefore:

$$\dot{S}_{mat} = \dot{S}_{kediss} + \dot{S}_{sens}. \quad (17)$$

The ratio

$$\alpha = \dot{S}_{sens} / \dot{S}_{kediss} \quad (18)$$

is a measure of the degree of irreversibility of the system, which is zero if all the production of entropy is due to the unavoidable dissipation of the mechanical energy (Lucarini et al., 2010). The parameter  $\alpha$  introduced above is related to the Bejan number  $\mathcal{Be}$  as  $\mathcal{Be} = \alpha + 1$  (Paoletti et al., 1989). Systems with large  $\alpha$  are instead characterized by high thermal dissipation relatively to the mechanical viscous dissipation and therefore by a higher degree of irreversibility.

## 5.2. Dissipative properties of circulation regimes

In this section we analyse the dissipative properties of the different circulations described in Sec. 4 as the parameters  $\Omega$  and  $\tau$ , and consequently  $\mathcal{Ro}$  and  $\mathcal{F}_f$ , are varied. Sensitivity studies of dissipative properties have been proposed first by Kunz et al. (2008) and then used extensively in Pascale et al. (2011b) and Boschi et al. (2012) as an insightful way to assess the models' tuning and their thermodynamical properties. In the following, we plot quantities in the  $(\Omega^*, \tau)$  plane for practical purposes, and we overplot the values of  $\log_{10} \mathcal{Ro}$  and  $\log_{10} \mathcal{F}_f$  (Fig. 4 to Fig. 11).

464 *Kinetic energy dissipation and meridional heat transport.* In Fig. 4, the  
465 results of the numerical simulations show that for  $10^{-2} < \mathcal{Ro} < 1$  and  
466  $1 < \mathcal{F}_f < 10^3$  there is the highest total dissipation of kinetic energy,  $D$ . We  
467 observe a non-trivial dependence on  $\Omega$  and  $\tau$ . The most intense dissipation  
468 is centered around  $\mathcal{Ro} \approx 0.1$  and  $\mathcal{F}_f \approx 10^2$  ( $\tau = 12$  hours and  $\Omega^* = 1$ ), with  
469  $D \approx 0.45 \text{ W m}^{-2}$ . This is mainly associated with the dissipation of kinetic  
470 energy in the boundary layer, as can be seen in Fig. 5 where  $D_{phys}$  is shown.  
471 On the base of the discussion in Section 4, we can speculate that at low val-  
472 ues of  $\Omega$ , the baroclinic eddies become larger than the size of the exoplanet  
473 (see equation (10) and related discussion) and thus do not develop; at high  
474 values of  $\Omega$  they become too small, convert inefficiently available potential  
475 energy into kinetic energy (Hunt, 1979), and dissipate quickly. Furthermore,  
476 the surface properties have a dramatic impact on the circulation, as shown  
477 also by James and Gray (1986), because the growth rate of the most unsta-  
478 ble baroclinic waves is strongly inhibited by horizontal shears (James, 1987)  
479 observed, for example, in Fig. 2(e). This explains the drop of  $D$  at high  
480  $\mathcal{F}_f$  and intermediate  $\mathcal{Ro}$ . On the other hand, strong drag leads to kinetic  
481 energy extraction early in the development of baroclinic eddies. Therefore,  
482 the optimal situation is expected for intermediate values of  $\Omega$  and surface  
483 drag. Our results are in agreement with those of Kleidon et al. (2003, 2006),  
484 who considered the case  $\Omega^* = 1$  only.

485 Moving on to fastly rotating planets, there is a significant decrease of  $D$   
486 at low thermal Rossby number ( $\mathcal{Ro} < 10^{-2}$ ) for any value of  $\mathcal{F}_f$  (zonostrophic  
487 flow, ZN). The strength of the Lorenz energy cycle therefore tends to become  
488 more insensitive to the surface properties. Interestingly, also circulations of  
489 slowly rotating planets with low drag ( $\mathcal{Ro} > 1$ ,  $\mathcal{F}_f > 10^4$ , corresponding  
490 with the super-rotation regime, see Fig.1(b)) have very weak kinetic energy  
491 dissipation. The dissipation rate remains high for slow rotation and for strong  
492 drag ( $\mathcal{F}_f \leq 0.1$ ,  $\mathcal{Ro} \geq 10$ , AS circulations, Fig.1(a)). This is consistent with  
493 the fact that in the low rotation, axisymmetric circulations, baroclinicity is  
494 mostly absent, and the dissipation of kinetic energy is simply related to the  
495 strength of the surface drag, which extracts kinetic energy from the mean  
496 flow, thus causing very weak winds near the surface.

497 The meridional heat transport (Peixoto et al., 1991) is in general a very  
498 important quantity in planetary atmospheres (Lorenz et al., 2001) and it is  
499 associated with the radiative imbalance between high and low temperature  
500 regions. The zonal mean of the meridional heat transport  $T(\vartheta)$  is worked  
501 out at each latitude  $\vartheta$  by integrating the longitudinally averaged top-of-the-

atmosphere (TOA) radiation budget (Lucarini and Ragone, 2011). A scalar index,  $MHT$ , of the meridional heat transport is then defined as half of the difference of the values of the poleward heat transport in the two hemispheres at  $30^\circ$  latitude,

$$MHT = 1/2(Tr(\pi/3) - Tr(-\pi/3)). \quad (19)$$

$MHT$  thus represents the net heat flowing out of the equatorial region through zonal walls placed at  $30^\circ$ .

Overall we observe that the meridional heat transport increases with  $Ro$ , in agreement with the results found in Vallis and Farneti (2009). This general feature is due to the inefficiency of the too small baroclinic eddies at high  $\Omega$  in transporting heat (eq. 10).

Furthermore, it is evident that for intermediate rotation rates ( $1/5 \leq \Omega^* \leq 2$ )  $MHT$  peaks at  $\tau \approx 1$  day ( $\approx 1$  PW), that is in the region of baroclinic circulations (Fig. 1(b) and 1(a)), coinciding with the maximum in dissipation (Fig. 4). It is well known in fact that midlatitude eddies constitute a very important mechanism of meridional heat transport (Lorenz, 1967; James, 1994). This is also clear from the zonal mean of the transient eddy flux  $\overline{v'T'}$  (not shown), which reaches the highest values  $\approx 8 \text{ K ms}^{-1}$  at 900 hPa and 50 N/S for the values of  $\tau$  maximizing  $D$ , compared to  $0.5 \text{ K ms}^{-1}$  for  $\tau = 45$  min (at 700 hPa and 60 N/S) and  $4 \text{ K ms}^{-1}$  for  $\tau = 500$  days (at 1000 hPa and 50 N/S). Just for the sake of comparison, let us note that for earth's circulation  $\overline{v'T'}|_{max} \approx 15 \text{ K ms}^{-1}$  at 850 hPa and 50 N/S (e.g. James, 1994). In the slow rotation region ( $Ro \approx 10$ ) we have the largest heat transport ( $\approx 1.5$  PW) at high drag ( $\tau$  of few hours), which may be explained by lower wind velocities in the lower branch of the Hadley cell (equatorwards motion).

*Efficiency and material entropy production.* The efficiency diagram (Fig. 7) shows that the highest value of  $\eta$  lay in the intermediate rotation range with values of  $\approx 3\%$  in correspondence of the baroclinic and axisymmetric circulations. At low rotations, the high-drag circulations ( $\mathcal{F}_f < 1$ ) are the most efficient. Interestingly, we note that circulations tending toward equatorial super-rotation have a quite substantial drop in efficiency which reduces to  $\approx 1\%$ . At low  $Ro$  the thermodynamic efficiency drops below  $1\%$  because of the drastic drop in  $D$  associated with the weakening of the Lorenz energy cycle, therefore zonostrophic flows are very inefficient circulation regimes in terms of converting heat into mechanical work. Let us note that although we are dealing with a dry atmosphere, and therefore very different from a moist one (in which the magnitude of the heat losses and gain is much higher,

for example the latent heat gives a positive heating contribution of  $\sim 80$   $\text{W m}^{-2}$ ),  $\eta$  has comparable values (see e.g. Lucarini et al., 2010) and does not generally exceeds 3%.

The material entropy production terms (eq. (14, 16 and 17)) are shown in Fig. 8-10. Fig. 8 shows the contribution due to thermal dissipation  $\dot{S}_{sens}$  (15). This is dominated by  $\dot{S}_F$ , which accounts for almost 2/3 of  $\dot{S}_{sens}$  and is almost independent from  $Ro$ , having its highest values for  $\tau \sim 3$  days. Such a pattern is explained by a trade-off mechanism between the sensible heat flux, which decreases with  $\tau$  independently at any  $Ro$  (not shown), and the temperature difference between the surface and the near-surface atmosphere, which increases with  $\tau$  since, due to eq. (7), surface and atmospheres tend to be more decoupled. The entropy production associated with the dissipation of kinetic energy,  $\dot{S}_{kedis}$  (Fig. 9) closely follows the pattern of  $D$  (Fig. 4) as evident from its own definition (eq. (14)).

The total material entropy production (17) is the sum of the two, so its properties are determined mainly by  $\dot{S}_{sens}$  which is generally larger than  $\dot{S}_{kedis}$  ( $\sim 1 - 2$  times in the at low-intermediate rotation rates, as can be seen in Fig. 11 where the irreversibility parameter  $\alpha$  is shown, and up to 10 times for fast rotating planets). The region of highest material entropy production ( $\approx 3.5 \text{ mW m}^{-2} \text{ K}^{-1}$ ) is observed for  $0.1 \leq Ro \leq 0.01$  and  $10^2 \leq \mathcal{F}_f \leq 10^3$ , and generally the whole region of the diagram in Fig. 1(b) with  $0.5 \text{ day} \leq \tau \leq 5 \text{ days}$  have large material entropy production. Overall, the material entropy production tends to be fairly low ( $\approx 1.5 \text{ mW m}^{-2} \text{ K}^{-1}$ ) for fast rotation speeds (e.g.  $Ro \sim 10^{-3}$ ) where we have very low values of  $\dot{S}_{sense}$  and lower values of  $\dot{S}_{kedis}$ . Let us note that the portion of the diagram corresponding to super-rotating fluids (SR in Fig. 1(a)) is characterized by very low mechanical and thermal dissipation and therefore very low material entropy production. In this respect super-rotating flows are quite interesting since such circulations are also characterized by very low efficiency. In other terms they seem to have a behavior close to inviscid, non-dissipative fluids (for which  $D = 0$  and  $\dot{S}_{sens} = 0$  by definition). Mitchell and Vallis (2010) also pointed out some peculiar dynamical properties of super-rotating flows, as for example the fact that the equatorial, strong eastwards jet, once established, do not need eddy-forcing to be maintained. Interestingly, these results make clear that there is no obvious correspondence between the presence of large amount of kinetic energy in the atmosphere and the presence of an intense Lorenz energy cycle to support its generation. This matter has been hotly debated in a rather different scientific context, where the possibility of extracting

576 massive amounts of energy from the atmospheric circulation by wind turbines  
 577 is discussed (Miller et al., 2011).

578 A schematic diagram summarising the main thermodynamical properties  
 579 discussed so far for the different circulation regimes is shown in Fig. 1(b):

- 580 1. Baroclinic regime (BC): high  $D$ , high  $\eta$ , relatively high  $MHT$ ;
- 581 2. Super-rotation (SR): low  $D$ , low  $\eta$ , low  $\dot{S}_{mat}$ ;
- 582 3. Zonostrophic flow (ZN): low  $D$ , low  $MHT$ , low  $\eta$ ;
- 583 4. Axisymmetric flow (AS): high  $MHT$  and  $D$  for  $Ro > 1$ , high  $\eta$  for  
 584  $1 < Ro < 0.1$ , low  $D$ ,  $MHT$  and  $\eta$  for  $Ro < 0.01$ .

### 585 5.3. Implications for the Maximum Entropy Production Principle

586 In this section we briefly describe our results in the context of the Max-  
 587 imum Entropy Production Principle (MEPP, Paltridge, 1975, 1978, 2001),  
 588 as this conjecture has gained some momentum also in the planetary science  
 589 community (Lorenz et al., 2001; Taylor, 2010). MEPP has been used as  
 590 a closure condition for climatic toy-models (Lorenz et al., 2001) or simple  
 591 energy balance climate models (e.g. Paltridge, 1975) in order to determine  
 592 dynamical quantities as the meridional heat transport. A further, possible  
 593 application was shown by Kleidon et al. (2003) and Kunz et al. (2008), who  
 594 suggested to use MEPP as a guide for tuning sub-grid motion parameters of  
 595 PUMA, an atmospheric general circulation models (Fraedrich et al., 2005).  
 596 For example, let us consider the Rayleigh drag constant  $\tau$  (eq. 6 and following  
 597 discussion) depends on the drag coefficient  $\gamma_h$  which in turn depends on both  
 598 surface roughness and dynamical quantities. Therefore different values of  $\tau$   
 599 can be thought of associated with either different surface properties (as done  
 600 in the rest of the paper) or to different strengths of the turbulent transfer in  
 601 the planetary boundary layer. Following the second interpretation, Kleidon  
 602 et al. (2003) showed that the value of  $\tau$  giving the most realistic atmospheric  
 603 state was that maximizing the entropy production of the system. However,  
 604 one major criticism that MEPP has encountered is that it does not take into  
 605 account the effects of the rotation speed (Rodgers, 1976; Goody, 2007; Jupp  
 606 and Cox, 2010). This was related to the criticisms on whether one could  
 607 use MEPP to infer the meridional energy transport. In this study we are  
 608 in a position to have a broader look on the results of Kleidon et al. (2003)  
 609 since a more detailed diagnostics for the dissipative properties and a larger

610 dynamical range for atmospheric circulations are available. Of course our  
 611 aim is not, and we do not claim, to prove or disprove MEPP, for which a  
 612 rigorous demonstration is still missing (Dewar, 2005; Grinstein and Linsker,  
 613 2007).

614 In order to test MEPP, we perform control runs in which the full bound-  
 615 ary layer scheme (Louis, 1979; Louis et al., 1981) is employed without the  
 616 simplification of Sect. 3.2 (so  $\tau$  is not prescribed but dynamically determined  
 617 depending on the winds and vertical stability). In the following we shall re-  
 618 fer to them and to quantities evaluated for such simulations with the label  
 619 “BLS” (boundary layer scheme). In BLS simulations the drag coefficient is  
 620 consistently determined at each timestep and each grid-point according to the  
 621 Monin-Obukhov theory (e.g. Arya, 1988) and not prescribed as a constant  
 622 parameter. Since this set up employs a more refined and realistic represen-  
 623 tation of the boundary layer physics, we consider it as our “reality” towards  
 624 which comparing simulations in which the rougher, tunable  $\tau$ -scheme is used.  
 625 Zonal means of the BLS simulations are shown in Fig. 13 – cross sections of  
 626 temperature and zonal winds – and in Fig.14 – meridional streamfunctions –  
 627 for simulations for  $\Omega^* = 1/10, 1, 8$  respectively. For each  $\Omega^*$ , we consider  $\tau$  as  
 628 a tunable parameter and select the value  $\tau_{max}(\Omega^*)$  maximising  $\dot{S}_{mat}$  (which  
 629 can be easily visualized in Fig. 10). Furthermore, we take into account also  
 630  $\dot{S}_{kedis}$  (Fig. 9), so that we can be informative also on the maximum dissipa-  
 631 tion principle (Lorenz, 1967; Ozawa et al., 2003; Schulman, 1977; Pascale  
 632 et al., 2011b). We denote with  $\tilde{\tau}_{max}(\Omega^*)$  the values of  $\tau$  maximising  $\dot{S}_{kedis}$ .  
 633 As can be seen in Fig. 9-10,  $\tau_{max}$  and  $\tilde{\tau}_{max}$  differ mostly for  $\Omega^* \leq 1/2$  (where  
 634 the maximum dissipation steady states occur for  $\tau$  of few hours) whereas  
 635 they are mostly the same (1 day) for  $\Omega^* > 2$  days ( $\tau \approx 1$  day).

636 In Fig. 12(a) and 12(b) we compare  $\dot{S}_{mat}(\Omega^*; \tau_{max})$  and  $\dot{S}_{kedis}(\Omega^*; \tau_{max})$   
 637 (dashed line) with  $\dot{S}_{mat}^{BLS}(\Omega^*)$  and  $\dot{S}_{kedis}^{BLS}(\Omega^*)$  respectively (continuous lines).  
 638 On the same diagrams we also show the same quantities for  $\tau = 0.1 \tau_{max}(\Omega^*)$   
 639 (dotted line) and  $\tau = 10 \tau_{max}(\Omega^*)$  (dotted-dashed line) in order to provide  
 640 an indication of the sensitivity of  $\dot{S}_{mat}$  and  $\dot{S}_{kedis}$  with respect to  $\tau_{max}$ . The  
 641 MEPP estimate of  $\dot{S}_{mat}$  slightly overestimate the values obtained in controls  
 642 runs ( $\leq 5\%$ ) but, impressively, captures fairly well the dependence on  $\Omega^*$ .  
 643 Similarly, the values of  $\dot{S}_{kedis}$  obtained for  $\tau_{max}$  compare relatively well with  
 644 the ones obtained in the controls runs. Circulations corresponding to  $\tau_{max}$   
 645 are indeed fairly similar to BLS circulations, as can be seen by comparing  
 646 Fig. 13(a,b,c) with Fig. 2(b,e,h) and Fig. 14(a,b,c) with Fig. 3(b,e,h).

647 When the values of  $\tilde{\tau}_{max}(\Omega^*)$  associated with the maximum of  $\dot{S}_{kedis}$  is

648 instead taken into account (Fig. 12(c)-12(d)), we observe that  $\dot{S}_{mat}(\Omega^*, \tilde{\tau}_{max})$   
649 provides again a quite good estimate of  $\dot{S}_{mat}^{BLS}$ , with a slight underestimate  
650 ( $\approx 9\%$ ) for  $\Omega^* < 1/2$ , due to the fact that for such values of the rotation  
651 rate  $\tilde{\tau}_{max}$  bends towards smaller  $\tau$  where  $\dot{S}_{mat}$  tends to decrease (Fig. 10).  
652 More unsatisfactory is  $\dot{S}_{kediss}(\Omega^*, \tilde{\tau}_{max})$  again for  $\Omega^* < 1/2$ , with a difference  
653 of about 16% with respect to  $\dot{S}_{kediss}^{BLS}$ .

654 In the end, both maximum entropy production and maximum dissipa-  
655 tion principle provide fairly reasonable estimates of  $\dot{S}_{kediss}^{BLS}$  and  $\dot{S}_{mat}^{BLS}$ , with  
656 the maximum entropy production one having better skills at low  $\Omega^*$ . The  
657 quasi-equivalence of the the two methods is due to the fact that, for such  
658 simulations, both  $\dot{S}_{mat}$  and  $\dot{S}_{kediss}$  have their maxima in the  $(\Omega^*, \tau)$  almost in  
659 the same regions. These results seem to confirm, in a relatively large range  
660 of dynamical regimes, the possibility of using MEPP in its weak form, as a a  
661 guide for tuning sub grid parameters associated with turbulent motions, as  
662 indicated by Kleidon et al. (2003).

## 663 6. Conclusions

664 Stimulated by the ongoing development of exoplanet sciences, in this  
665 study we have investigated the nonequilibrium thermodynamic properties  
666 (kinetic energy dissipation, material entropy production, efficiency, merid-  
667 ional heat transport) of optically-thin, non-condensing planetary atmospheres  
668 at different values of the thermal Rossby number  $\mathcal{Ro}$  and the Taylor number  
669  $\mathcal{F}_f$  through a systematic variation of the rotation rate  $\Omega$  and surface drag  
670 time constant  $\tau$ . The most relevant achievement of this study has been the  
671 characterization of the nonequilibrium properties of the different circulation  
672 regimes (axisymmetric, super-rotation, baroclinic, barotropic, zonostrophic)  
673 obtained with numerical simulations with some interesting connection to the  
674 Maximum Entropy Production Principle (MEPP).

675 Slowly rotating planets ( $\mathcal{Ro} > 1$ ) circulation are mostly Hadley cell-  
676 dominated but tend to equator; super-rotation for  $\mathcal{F}_f > 10^5$ . For interme-  
677 diate rotation rates ( $1 < \mathcal{Ro} < 0.01$ ) an axisymmetric ( $\mathcal{F}_f < 10$ ), baroclinic  
678 ( $10 < \mathcal{F}_f < 10^5$ ) and barotropic ( $\mathcal{F}_f > 10^5$ ) regime are found. At high  
679 rotation rates ( $\mathcal{Ro} < 0.01$ ) circulations are characterized by multiple jets  
680 (zonostrophic) for  $\mathcal{F}_f > 10^4$ .

681 The baroclinic regime has high values of  $D$  and  $MHT$  since midlatitude  
682 baroclinic waves provide a very effective way to convert available potential  
683 energy into mechanical kinetic energy and transport energy from low to high

latitudes. Such mechanism is inhibited by strong barotropic shears characterizing the barotropic regime and therefore both  $D$  and  $MHT$  experience lower values. The axisymmetric regime has different thermodynamic properties depending on the value of  $\mathcal{Ro}$  at which it is realised. For  $\mathcal{Ro} > 1$ , a very intense Hadley cell develops associated with high  $MHT$  and  $D$ ; for  $1 < \mathcal{Ro} < 0.1$  such quantities are weaker but circulations are more efficient in converting heat into mechanical work (high  $\eta$ ); at faster rotation speeds ( $\mathcal{Ro} < 0.01$ ) a dramatic drop in  $D$ ,  $MHT$  and  $\eta$  is observed. A very interesting case is that of circulation approaching equatorial super-rotation ( $\mathcal{Ro} \leq 10$ ,  $\mathcal{F}_f > 10^5$ ), for which low  $D$ , low  $\eta$ , low  $\dot{S}_{mat}$  occurs, thus showing a behavior close to inviscid, non-dissipative fluids (for which  $D = 0$  and  $\dot{S}_{sens} = 0$  by definition). Zonostrophic flows low, typical of fast rotating, low surface drag planets, have a very weak atmospheric energy cycle (low  $D$ ), are very inefficient in converting potential energy into work and have very low meridional heat transport  $MHT$ , therefore showing a temperature profile close to the radiative-convective equilibrium (which by definition has  $MHT = 0$ ).

The thermal dissipation  $\dot{S}_{sens}$  is instead fairly insensitive to  $\mathcal{Ro}$  and is determined mainly by the timeconstant  $\tau$ , due to a trade-off mechanism between the temperature difference and the heat flux.

Moreover, we have shown that the possibility of applying MEPP in its weak form, e.g. as a tool for providing guidance in tuning subgrid scale, seems to work relatively well in the range of values of the rotation rate considered in this study, thus extending the results obtained by Kleidon et al. (2003) when considering the terrestrial rotation rate only. Interestingly, there is broad agreement between what prescribed by applying MEPP and the maximum dissipation principle.

This is a first preliminary study for a special case of dry atmosphere. The presence of the hydrological cycle has a huge effect on the circulation and on the energetics and would be definitely worth investigating. Another issue is the role of the surface heat capacity, which would also deserve a systematic investigation. Furthermore, thermodynamic and dynamical properties of slowly rotating planets, e.g. from  $\Omega^* = 1/10$  up to phase-locked planets, are still poorly known and would deserve more investigation too.

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724 Burrows, A. et al., 1997. A nongray theory of extrasolar giant planets and  
725 brown dwarfs. *The Astrophysical Journal* **491**, 856–875.

726 Arya, S. P., 1988. *Introduction to Micrometeorology*. Accademic Press.

727 Donohoe, A. and Battisti D. S., 2012. What determines meridional heat  
728 transport in climate models? *Journal of Climate* **25**, 3832 – 3850.

729 Becker, E., 2001. Symmetric stress tensor formulation of horizontal momen-  
730 tum diffusion in a global model of atmospheric circulation. *Journal of At-  
731 mospheric Sciences* **58**, 269–282.

732 Berger, A., 1978. Long-term variations of daily insolation and quaternary  
733 climatic change. *J. Atmos. Sci.* **35**, 2362-2367

734 Bonfils, X. et al., 2012. The HARPS search for southern extra-solar planets  
735 XXXI. the m-dwarf sample. *Astronomy and Astrophysics*, in press.

736 Boruck, W. and coauthors, 2011. Characteristics of Planetary Candidates  
737 Observed by Kepler. II. Analysis of the First Four Months of Data *The  
738 Astrophysical Journal* **736**, 19.

739 Boschi, R., Lucarini, V., Pascale, S., 2012. Bistability of the cli-  
740 mate around the habitable zone: a thermodynamic investigation.  
741 <http://arxiv.org/abs/1207.1254>.

742 Budyko, M. I., 1969. The effect of solar radiation variations on the climate  
743 of the Earth. *Tellus* **5**, 611-619.

744 Caballero et al., 2008. Axisymmetric, nearly inviscid circulations in non-  
745 condensing radiative-convective atmospheres. *Quarterly Journal of the  
746 Royal Meteorological Society* **134**, 1269–1285.

747 Charbonneau et al., 2009. A super-Earth transiting a nearby low-mass star.  
748 *Nature* **462**, 891–894.

- 749 Clancy et al., 2007. Dynamics of the venus upper atmosphere: global-  
750 temporal distribution of winds, temperature, and CO at the Venus  
751 mesopause. *Bull. Am. Astron. Soc.* **39**, 539.
- 752 Dahms, E., Lunkeit, F., Fraedrich, K., 2012. Low-frequency climate variabil-  
753 ity of an aquaplanet. *J. Clim.*, submitted.
- 754 DeGroot, S., Mazur, P., 1984. *Non-equilibrium thermodynamics*. Dover.
- 755 Dewar, R. C., 2005. Maximum entropy production and the fluctuation theo-  
756 rem. *Journal of Physics A* **38**, L371–L381.
- 757 Dobbs-Dixon, I., Agol, E., and Burrows, A., 2012. The Impact of Circum-  
758 planetary Jets on Transit Spectra and Timing Offsets for Hot Jupiters. *The*  
759 *Astrophysical Journal* **751**, 87
- 760 Dvorak, R., 2008. *Extrasolar Planets*. Wiley-VHC.
- 761 Eady, E., 1949. Long waves and cyclone waves. *Tellus* **1**, 33–52.
- 762 Egger, J., 1999. Numerical generation of entropies. *Monthly Weather Review*  
763 **127**, 2211–2216.
- 764 Eliassen, E., Machenhauer, B., Rasmussen, E., 1970. On a numerical method  
765 for integration of the hydrodynamical equations with a spectral representa-  
766 tion of the horizontal fields. Report no. 2, Inst. of Theor. Met., University  
767 of Copenhagen.
- 768 Fermi, E., 1956. *Thermodynamics*. Dover.
- 769 Fraedrich, K., Jansen, H., Kirk, E., Luksch, U., Lunkeit, F., 2005. The planet  
770 simulator: towards a user friendly model. *Meteorologische Zeitschrift*  
771 **14** (3), 299–304.
- 772 Fraedrich, K., Lunkeit, F., 2008. Diagnosing the entropy budget of a climate  
773 model. *Tellus A* **60** (5), 921–931.
- 774 Fraedrich, K., E. Kirk, U. Luksch, and F. Lunkeit, 2005. The Portable Uni-  
775 versity Model of the Atmosphere (PUMA): Storm track dynamics and low  
776 frequency variability. *Meteorol. Zeitschrift* **14**, 735–745.

777 Frierson, D. M. W., Held I. M., Zurita-Gotor, P., 2006. A gray-radiation  
778 aquaplanet moist GCM. Part I: static stability and eddy scale. *J. Atmos.*  
779 *Sci.* **63**, 2548–2566

780 Gallavotti, G., 2006. *Encyclopedia of mathematical physics*. Elsevier, Ch.  
781 Nonequilibrium statistical mechanics (stationary): overview, pp. 530–539.

782 Geisler, J. E., Pitcher, E. J., Malone, R. C., 1983. Rotating-fluid experiments  
783 with an atmospheric general circulation model. *Journal of Geophysical*  
784 *Research* **88** (C14), 9706–9716.

785 Genio, A. D., Suozzo, R., 1987. A comparative study of rapidly and slowly  
786 rotating regimes in a terrestrial general circulation model. *J. Atmos. Sci.*  
787 **44**, 973–986.

788 Goody, R., 2000. Sources and sinks of climate entropy. *Quarterly Journal of*  
789 *the Royal Meteorological Society* **126**, 1953–1970.

790 Goody, R., 2007. Maximum entropy production in climate theory. *Journal of*  
791 *the Atmospheric Sciences* **64**, 2735–2739.

792 Grinstein, G., Linsker, R., 2007. Comments on a derivation and application  
793 of the maximum entropy production principle. *J. Phys A* **40**, 9717–9720.

794 Held, I. M., Hou, A. Y., 1980. Nonlinear axially symmetric circulations in a  
795 nearly inviscid atmosphere. *Journal of Atmospheric Sciences* **37**, 515–533.

796 Heng, K., 2012. The study of climates of alien worlds. *American Scientist*  
797 **100** (4), 334–341, arXiv: 1206.3640

798 Heng, K., Frierson, D. M., Phillipps, P. J., 2011a. Atmospheric circulation  
799 of tidally locked exoplanets: II dual-band radiative transfer and convec-  
800 tive adjustment. *Monthly Notices of the Royal Astronomical Society* **420**,  
801 2669–2696.

802 Heng, K., Menou, K., Phillips, P. J., 2011b. Atmospheric circulation of  
803 tidally locked exoplanets: a suite of benchmark tests for dynamical solvers.  
804 *Monthly Notices of the Royal Astronomical Society* **413**, 2380–2402.

805 Heng, K. and Vogt, S. S., 2011. Gliese 581g as a scaled-up version of Earth:  
806 atmospheric circulation simulations. *Mon. Not. R. Astron. Soc.* **415**, 2145.

- 807 Hide, R., 1953. Some experiments on thermal convection in a rotating liquid.  
808 Q. J. R. Meteorol. Soc. **79**, 161.
- 809 Hide, R., 1969. Some laboratory experiments on free thermal convection in  
810 a rotating fluid subject to a horizontal temperature gradient and their  
811 relation to the theory of the global atmospheric circulation. Corby, G.A.  
812 (Ed.), The Global Circulation of the Atmosphere, Royal Meteorological  
813 Society, London, 196–221.
- 814 Hide, R., 2010. A path of discovery in geophysics fluid dynamics. Astronomy  
815 and Geophysics **51** (4), 16–23.
- 816 Hide, R., Mason, P., 1975. Some experiments on thermal convection in a  
817 rotating liquid. Adv. Phys. **24**, 47–99.
- 818 Holton, J. R., 2004. An introduction to Dynamic Meteorology. Elsevier Aca-  
819 demic Press.
- 820 Hourdin, F., O. et al. 1995. Numerical simulations of the general circulations  
821 of the atmosphere of Titan. Icarus **117**, 358–374.
- 822 Hunt, B., 1979. The influence of the earth’s rotation rate on the general  
823 circulation of the atmosphere. J. Atmos. Sci. **36**, 1392–1408.
- 824 Jablonowski, C., Williamson, D., 2011. Numerical techniques for global at-  
825 mospheric models. Springer, Ch. The pros and cons of diffusion, filters and  
826 fixers in atmospheric general circulation models, pp. 381–493.
- 827 James, I., 1994. Introduction to Circulating Atmosphere. Cambridge Univer-  
828 sity Press.
- 829 James, I., Gray, L., 1986. Concerning the effect of surface drag on the circu-  
830 lation of a baroclinic planetary atmosphere. Quart. J. R. Met. Soc. **112**,  
831 1231–1250.
- 832 James, I. N., 1987. Suppression of baroclinic instability in horizontal sheared  
833 flows. Journal of Atmospheric Sciences **44** (24), 3710–3720.
- 834 Johnson, D., 1997. ”General coldness of climate” and the second law: Impli-  
835 cations for modelling the earth system. Journal of Climate **10**, 2826–2846.

- 836 Johnson, D. R., 2000. General Circulation Model Development: Past, Present  
837 and Future. Accademic Press, New York, Ch. Entropy, the Lorenz Energy  
838 Cycle and Climate, pp. 659–720.
- 839 Joshi, M. M., 2003. Climate model studies of synchronously rotating planets  
840 *Astrobiology* **3**(2), 415–27, PMID 14577888
- 841 Jupp, T., Cox, P., 2010. MEP and planetary climates: insights from a two-  
842 box climate model containing atmospheric dynamics. *Philosophical Trans-*  
843 *actions of the Royal Society B* **365**, 1355–1365.
- 844 Kleidon, A., 2009. Nonequilibrium thermodynamics and maximum entropy  
845 production in the earth system. *Naturwissenschaften* **96**, 653–677.
- 846 Kleidon, A., Fraedrich, K., Kirk, E., Lunkeit, F., 2006. Maximum en-  
847 tropy production and the strenght of boundary layer exchange in an at-  
848 mospheric general circulation model. *Geophysical Research Letters* **33**,  
849 doi:10.1029/2005GL025373.
- 850 Kleidon, A., Fraedrich, K., Kunz, T., Lunkeit, F., 2003. The atmospheric  
851 circulation and the states of maximum entropy production. *Geophysical*  
852 *Research Letters* **30** (23), doi:10.1029/2003GL018363.
- 853 Kleidon, A., Lorenz, R., 2005. Non-equilibrium Thermodynamics and the  
854 Production of Entropy. *Understanding Complex Systems*. Springer, Berlin.
- 855 Kondepudi, D., Prigogine, I., 1998. *Modern Thermodynamics: From Heat*  
856 *Engines to Dissipative Structure*. John Wiley, Hoboken, N.J.
- 857 Kundu, P., Cohen, I. M., 2004. *Fluid Mechanics*. Accademic Press.
- 858 Kunz, T., Fraedrich, K., Kirk, E., 2008. Optimisation of simplified GCMs  
859 using circulation indices and maximum entropy production. *Climate Dy-*  
860 *namics* **30**, 803–813.
- 861 Kuo, H., 1965. On formation and intensification of tropical cyclones through  
862 latent heat release by cumulus convection. *J. Atmos. Sci.* **22**, 40–63.
- 863 Kuo, H., 1974. Further studies of the parametrisation of the influence of  
864 cumulus convection on large-scale flow. *J. Atmos. Sci.* **31**, 1232–1240.

- 865 Lacis, A., Hansen, K., 1974. A parametrisation for the absorption of solar  
866 radiation in the earth's atmosphere. *J. Atmos. Sci.* **31**, 118–133.
- 867 Laursen, L., Eliassen, E., 1989. On the effect of the damping mechanisms in  
868 an atmospheric general circulation model. *Tellus* **41A**, 385–400.
- 869 Lewis N. K. et al., 2010. Atmospheric circulation of eccentric hot nep-  
870 tune GJ436b. *The Astrophysical Journal* **720**, 344, doi:10.1088/0004-  
871 637X/720/1/344.
- 872 Lorenz, E., 1955. Available potential energy and the maintenance of the  
873 general circulation. *Tellus* **7**, 271–281.
- 874 Lorenz, E., 1960. Generation of available potential energy and the intensity  
875 of the general circulation. Pergamon, Tarrytown, N.Y.
- 876 Lorenz, E., 1967. The nature and theory of the general circulation of the  
877 atmosphere. Vol. 218.TP.115. World Meteorological Organization.
- 878 Lorenz, R., Lunine, J., Withers, P., McKay, C., 2001. Titan, Mars and Earth:  
879 Entropy production by latitudinal heat transport. *Geophysical Research*  
880 *Letters* **28** (3), 415–418.
- 881 Louis, J., 1979. A parametric model of vertical eddy fluxes in the atmosphere.  
882 *Bound. Layer Meteorol.*, 187–202.
- 883 Louis, J., Tiedke, M., Geleyn, J., 1981. A short history of the pbl parametri-  
884 sation at ecmwf. *Proceedings of the ECMWF Workshop on Planetary*  
885 *Boundary Layer Parametrization*, 59–80.
- 886 Lucarini, V., 2009. Thermodynamic efficiency and entropy produc-  
887 tion in the climate system. *Physical Review E* **80**, 021118,  
888 doi:10.1103/PhysRevE.80.02118.
- 889 Lucarini, V., 2012. Modelling complexity: the case of climate science.  
890 arXiv:1106.1265v1 [physics.hist-ph]; in press, *Proceedings of the Confer-*  
891 *ence "Models, Simulation and the Reduction of Complexity"*, De Gruyter  
892 Verlag, Hamburg.
- 893 Lucarini, V., Ragone, F., 2011. Energetics of Climate Models: Net Energy  
894 Balance and Meridional Enthalpy Transport. *Reviews of Geophysics* **49**,  
895 RG1001, doi:10.1029/2009RG000323.

- 896 Lucarini, V., Fraedrich, K., Lunkeit, F., 2010. Thermodynamic analysis of  
897 snowball earth hysteresis experiment: efficiency, entropy production and  
898 irreversibility. *Quarterly Journal of Royal Meteorological Society* **136**, 1–  
899 11.
- 900 Lucarini, V., Fraedrich, K., Ragone, F., 2011. New results on the thermody-  
901 namic properties of the climate. *Journal of the Atmospheric Sciences* **68**,  
902 2438–2458.
- 903 Lunkeit, F., Borth, H., Böttinger, M., Fraedrich, K., Jansen, H., Kirk, E.,  
904 Kleidon, A., Luksch, U., Paiewonsky, P., Schubert, S., Sielmann, S., Wan,  
905 H., 2010. Planet simulator, reference manual (version 16). Tech. rep., Uni-  
906 versity of Hamburg, [www.mi.uni-hamburg.de/Downloads-un.245.0.html](http://www.mi.uni-hamburg.de/Downloads-un.245.0.html).
- 907 Menou, K., Rauscher, E., 2009. Atmospheric circulations of hot jupiters: a  
908 shallow three dimensional model. *The Astrophysical Journal* **700**, 887–897.
- 909 Merlis, T. M. and Schneider, T., 2010. Atmospheric dynamics of Earth-like  
910 tidally locked aquaplanets. *Journal of Advances in Modeling Earth Sys-*  
911 *tems*, **2** (13), 17 pp.
- 912 Miller L. M., Gans F., Kleidon A., 2011. Jet stream wind power as a re-  
913 newable energy resource: little power, big impacts. *Earth Syst. Dynam.* **2**,  
914 201–212.
- 915 Mitchell, J. L. and Vallis, G. K., 2010. The transition to superrotation in  
916 terrestrial atmospheres. *Journal of Geophysical Research*, **115** E12008,  
917 doi:10.1029/2010JE003587.
- 918 Navarra, A., Boccaletti, C., 2002. Numerical general circulation experiments  
919 of sensitivity to earth rotation rate. *Climate Dynamics* **19**, 467–483.
- 920 Orszag, S., 1970. Transform method for the calculation of vector coupled  
921 sums. *J. Atmos. Sci.*, 890–895.
- 922 Ozawa H, Ohmura A, Lorenz R. D., Pujol T., (2003). The second law of  
923 thermodynamics and the global climate system: A review of the maximum  
924 entropy production principle. *Reviews of Geophysics* **41** 1018, 2003 (doi:  
925 10.1029/2002RG000113)

- 926 Paltridge, G. W., 1975. Global dynamics and climate-a system of minimum  
927 entropy exchange. Quarterly Journal of the Royal Meteorological Society  
928 **101**, 475–484.
- 929 Paltridge, G. W., 1978. The steady state format of global climate. Quarterly  
930 Journal of Royal Meteorological Society **104**, 927–945.
- 931 Paltridge, G. W., 2001. A physical basis for a maximum of thermodynamic  
932 dissipation of the climate system. Quarterly Journal of the Royal Meteorological Society **127**, 305–313.
- 934 Paoletti, S., Rispoli, F., Sciubba, E., 1989. Calculation of exergetic losses in  
935 compact heat exchanger passages. ASME AES **10** (2), 21–29.
- 936 Pascale, S., Gregory, J., Ambaum, M., Tailleux, R., 2011a. Climate entropy  
937 budget of the HadCM3 atmosphere-ocean general circulation model and  
938 FAMOUS, its low-resolution version. Climate Dynamics **36** (5-6), 1189–  
939 1206.
- 940 Pascale, S., Gregory, J., Ambaum, M., Tailleux, R., 2011b. A parametric  
941 sensitivity study of entropy production and kinetic energy dissipation using the FAMOUS AOGCM. Climate Dynamics, doi 10.1007/s00382-011-0996-2.
- 944 Pauluis, O., Held, M., 2002a. Entropy budget of an atmosphere in radiative-convective equilibrium. Part I: Maximum work and frictional dissipation. Journal of the Atmospheric Sciences **59**, 125–139.
- 947 Pauluis, O., Held, M., 2002b. Entropy budget of an atmosphere in radiative-convective equilibrium. Part II: Latent heat transport and moist processes. Journal of the Atmospheric Sciences **59**, 140–149.
- 950 Peixoto, J., Oort, A., de Almeida, M., Tomé, A., 1991. Entropy budget of  
951 the atmosphere. Journal of Geophysical Research **96**, 10981–10988.
- 952 Peixoto, J. P., Oort, A., 1992. Physics of the Climate. Springer-Verlag, New  
953 York.
- 954 Pierrehumbert, R. T., A Palette of Climates for Gliese 581g. The Astrophysical Journal Letters **726** (1), L8.



956 Rauscher E. and Menou, K., 2012. The role of drag in the energet-  
957 ics of strongly forced exoplanets. *The Astrophysical Journal* **745**, 78  
958 doi:10.1088/0004-637X/745/1/78

959 Read, P., 2001. Transition to geostrophic turbulence in the laboratory, and  
960 as a paradigm in atmospheres. *Surveys Geophys.* **22**, 231–249.

961 Read, P., 2011. Dynamic and circulation regimes of terrestrial planets. *Plan-  
962 etary and space sciences* **59**, 900–914.

963 Read, P., Collins, M., Früh, W.-G., Lewis, S., Lovegrove, A., 1998. Wave  
964 interactions and baroclinic chaos: a paradigm for long timescale variability  
965 in planetary atmospheres. *Chaos Solitons Fractals* **9**, 1221–1227.

966 Adams, D., K., Rennó, N., O., 2005. Thermodynamic efficiencies of an ide-  
967 alized global climate model. *Climate Dynamics* **25**, 801–813.

968 Rodgers, C., 1976. Minimum entropy exchange principle-reply. *Quarterly  
969 Journal of Royal Meteorological Society* **102**, 455–457.

970 Sasamori, T., 1968. The radiative cooling calculation for application to gen-  
971 eral circulation experiments. *J. Appl. Meteorol.* **7**, 721–729.

972 Schulman, L. L., 1977. A theoretical study of the efficiency of the general  
973 circulation, *J. Atmos. Sci.*, **34**, 559580.

974 Seager, S. and Deming, D., 2010. Exoplanet atmospheres. *Annual Review of  
975 Astronomy and Astrophysics.* **48**, 631–672.

976 Sellers, W. D. 1969. A global climatic model based on the energy balance of  
977 the Earth-Atmosphere system. *J. Appl. Meteor.* **8**, 392–400.

978 Showman, A., Cho, J.-K., Menou, K., 2010. Atmospheric Circulation of Ex-  
979 oplanets. Invited review for the book "Exoplanets". S. Seager Eds., Univ.  
980 Arizona Press, pp 471–516.

981 Showman, A. P., Fortney, J. J., Lian, Y., Marley, M. S., Freedman, R. S.,  
982 Knutson, H. A., Charbonneau, D., 2009. Atmospheric circulation of hot  
983 jupiters: Coupled radiative-dynamical general circulation model simula-  
984 tions of HD 189733b and HD 209458b. *The Astrophysical Journal* **699**,  
985 564–584.

986 Slingo, A., Slingo, J., 1991. Response of the national center for atmospheric  
987 research community climate model to improvements in the representation  
988 of clouds. *J. Geophys. Res.* **96**, 341–357.

989 Stephens, G., 1978. Radiation profiles in extended water clouds. II:  
990 parametrization schemes. *J. Atmos. Sci.* **35**, 2123–2132.

991 Stephens, G., Ackermann, S., Smith, E., 1982. A shortwave parametrization  
992 scheme. *J. Atmos. Sci.* **41**, 687–690.

993 Taylor, F., 2010. Planetary atmospheres. Oxford University Press.

994 Thrastarson H. T. and Cho J. Y-K. (2011) Relaxation Time and Dissipation  
995 Interaction in Hot Planet Atmospheric Flow Simulations. *The Astrophysical Journal* **729**, 117.

997 Udry, S., Santos, N. C., 2007. Statistical properties of exoplanets. *Annual*  
998 *Review of Astronomy and Astrophysics* **45**, 397–439

999 Valencia, V., Sasselov, D. D., O’Connell, R., 2007. Radius and structure  
1000 models of the first super-earth planet. *The Astrophysical Journal* **656** (1),  
1001 545–551.

1002 Vallis, G. K., 2006. Atmospheric and Oceanic Fluid Dynamics. Cambridge  
1003 University Press.

1004 Vallis, G. K., Farneti, R., 2009. Meridional energy transport in the coupled  
1005 atmosphere-ocean system: scaling and numerical experiments. *Quarterly*  
1006 *Journal of Royal Meteorological Society* **135**, 1643–1660.

1007 Wang, Y. and Read, P., 2012. Diversity of planetary atmospheric circulations  
1008 and climates in a simplified general circulation model. *Proceedings IAU*  
1009 *Symposium No. 293*, 2012

1010 Williams, D., Pollard, D., 2002. Earth-like worlds on eccentric orbits: ex-  
1011 cursions beyond the habitable zone. *International Journal of Astrobiology*  
1012 **1** (1), 61–69.

1013 Williams, D., Pollard, D., 2003. Extraordinary climates of Earth-like planets:  
1014 three-dimensional climate simulations at extreme obliquity. *International*  
1015 *Journal of Astrobiology* **2** (1), 1–19.

- 1016 Williams, G., P., 1988a. The dynamical range of global circulations - I. Cli-  
1017 mate Dynamics **2**, 205–260.
- 1018 Williams, G. P., 1988b. The dynamical range of global circulations - II. Cli-  
1019 mate Dynamics **3**, 45–84.
- 1020 Williams, G., P., 1978. Planetary circulation. I: Barotropic representation of  
1021 jovian and terrestrial turbulence. J. Atmos. Sci. **35**, 1399–1426.
- 1022 Woollings, T., Thuburn, J., 2006. Entropy sources in a dynamical core atmo-  
1023 sphere model. Quarterly Journal of the Royal Meteorological Society **132**,  
1024 43–59.
- 1025 Wordsworth, R., Forget, F., Selsis, F., Millour, E., Charnay, B., Madeleine,  
1026 J.-B., 2011. Gliese 581d is the First Discovered Terrestrial-mass Exoplanet  
1027 in the Habitable Zone. The Astrophysical Journal Letters **733**, Issue 2,  
1028 doi: 10.1088/2041–8205/733/2/L48.
- 1029 Wordsworth, R., Read, P., Yamazaki, Y., 2008. Turbulence, waves and jets in  
1030 a differential heated rotating annulus experiment. Phys. Fluids **20**, 126602,  
1031 doi:10.1063/1.2990042.

Table 1: Parameters and symbols list

parameter/symbol	explanation	value
$\Omega_E$	Earth's rotation rate	$7.29 \cdot 10^{-5} \text{ rad}^{-1}$
$c_d$	specific heat of dry air	$1004 \text{ J kg}^{-1} \text{ K}^{-1}$
$c_{pw}$	specific heat of mixed layer model	$4180 \text{ J kg}^{-1} \text{ K}^{-1}$
$g$	gravitational acceleration	$9.81 \text{ m s}^{-2}$
$\rho_w$	ocean water density	$1030 \text{ kg m}^3$
$h_{ml}$	mixed layer depth	5 m
$C_{slab}$	slab-ocean areal heat capacity	$10^{-7} \text{ J K}^{-1} \text{ m}^{-2}$
$\alpha_s$	surface albedo	0.2
$S_0$	solar constant	$1365 \text{ W m}^{-2}$
$a$	planet's radius	6300 km
$\mathcal{R}o$	thermal Rossby number	
$\mathcal{F}_f$	"frictional" Taylor number	
$ASR$	absorbed stellar radiation at TOA	
$OLR$	outgoing long wave radiation at TOA	
$F_T$	surface sensible heat flux	
$F_{SW}^{toa}$		
$F_{SW}^{surf}$		
$F_{LW}^-$		
$\gamma_h$	heat transfer coefficient	
$\gamma_D$	drag coefficient	
$MHT$	meridional heat transport index	
$L_R$	Rossby deformation radius	
$N$	buoyancy frequency	
$\alpha$	irreversibility parameter	

## Figures' captions

- Figure 1
 

1(a) Schematic diagram of the  $(\mathcal{F}_f, \mathcal{R}o)$  parametric space spanned in this study. Overplotted are the values of  $\Omega^*$  (dashed-dotted) and  $\tau$  (dotted). We have schematically sketched the boundaries between different circulation regimes found for dry PlaSim on the base of the circulations (AS, axisymmetric; BC, baroclinic; BT, barotropic; ZN, zonostrophic; SR, super-rotation). Circles, pentagons and triangles represent the simulations performed with  $\Omega^* = 0.1, 1, 8$  respectively (see Fig.3 and 2). 1(b) The same regime diagram is summarizing schematically the properties of kinetic energy dissipation (continuous line, high and low  $D$ ), meridional energy transport (dotted-dashed line, high MHT), thermal material entropy production (dotted line, high and low  $\dot{S}_{sense}$ ), efficiency (dashed line, high and low  $\eta$ ).
- Figure 2
 

Zonal winds and temperature for  $\Omega^* = 1/10$  ( $\tau = 2700s$  (a), 1 day (b), 500 days (c)),  $\Omega^* = 1$  ( $\tau = 2700s$  (d), 1 days (e), 500 days (f)),  $\Omega^* = 8$  ( $\tau = 2700s$  (g), 1 days (h), 500 days (i)).
- Figure 3
 

As in Fig.3 but for the meridional mass streamfunction (units  $10^9 \text{ Kg s}^{-1}$ ).
- Figure 4
 

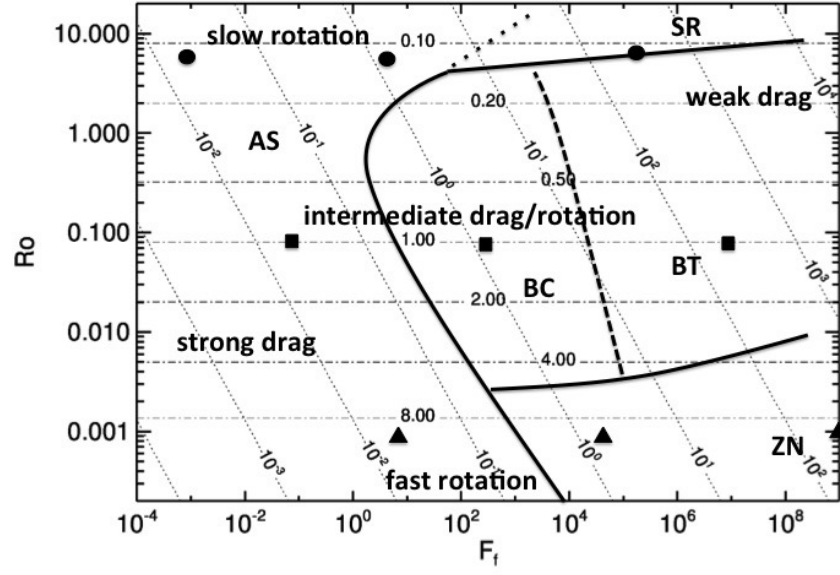
Total kinetic energy dissipation; overplotted (as in all the following plots) are the values of  $\log_{10} \mathcal{R}o$  (dashed) and  $\log_{10} \mathcal{F}_f$  (dotted).
- Figure 5
 

Contribution to the total kinetic energy dissipation due to parametrizations representing boundary layer stresses and gravity wave drag,  $D_{phys}$ .
- Figure 6
 

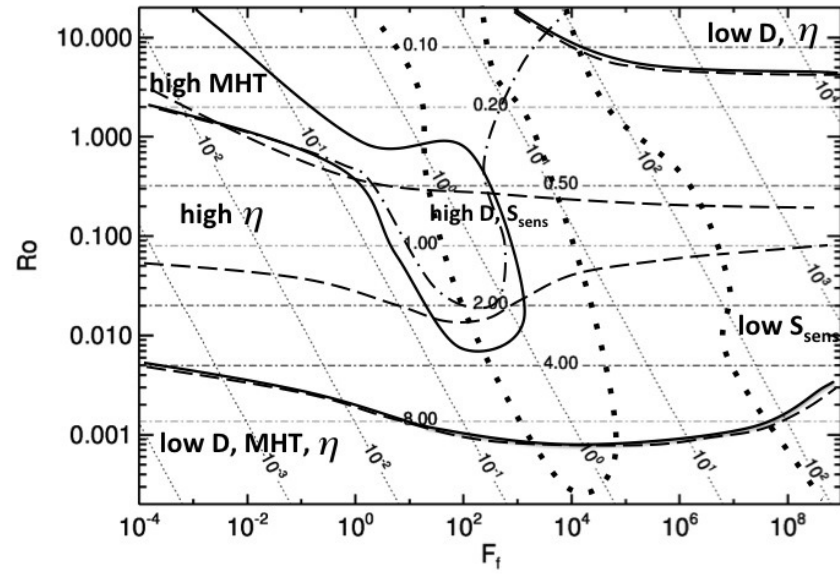
Atmospheric meridional energy transport index  $MHT$ .
- Figure 7
 

Carnot efficiency  $\eta$ .

- 1063     • Figure 8  
1064         Entropy production associated with surface sensible heat flux. Units  
1065         in  $10^{-3} \text{ W m}^{-2} \text{ K}^{-1}$ .
  
- 1066     • Figure 9  
1067         Material entropy production associated with dissipation of kinetic en-  
1068         ergy. Units in  $10^{-3} \text{ W m}^{-2} \text{ K}^{-1}$ .
  
- 1069     • Figure 10  
1070         Total material entropy production. Units in  $10^{-3} \text{ W m}^{-2} \text{ K}^{-1}$ .
  
- 1071     • Figure 11  
1072         Irreversibility parameter  $\alpha$ .
  
- 1073     • Figure 12  
1074          $\dot{S}_{mat}$  (12(a)) and  $\dot{S}_{kedis}$  (12(b)) for the control runs BLS (continuous  
1075         line), for  $\tau_{max}(\Omega^*)$  maximizing  $\dot{S}_{mat}$  (dashed) and for  $\tau = 0.1 \tau_{max}$  (dot-  
1076         ted) and  $\tau = 10 \tau_{max}$  (dotted-dashed) days. 12(c)-12(d) Same as in Fig.  
1077         12(a) and 12(b) but for  $\tilde{\tau}_{max}$  maximising  $\dot{S}_{kedis}$ .
  
- 1078     • Figure 13  
1079         Zonal winds and temperature for  $\Omega^* = 1/10$  (a),  $\Omega^* = 1$  (b) and  $\Omega^* = 8$   
1080         for the BLS simulations.
  
- 1081     • Figure 14  
1082         Meridional streamfunction for  $\Omega^* = 1/10$  (a),  $\Omega^* = 1$  (b) and  $\Omega^* = 8$   
1083         for the BLS simulations.

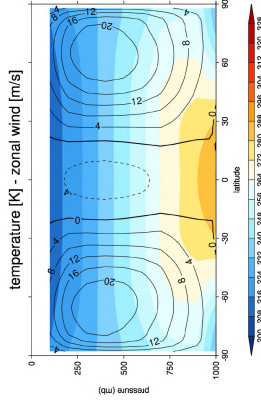


(a)

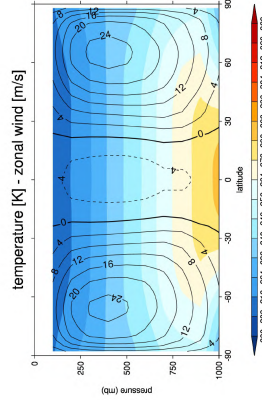


(b)

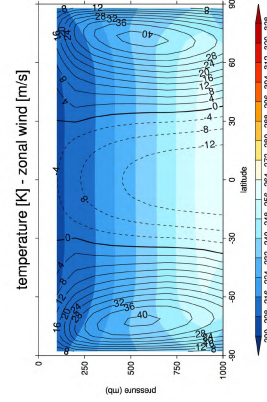
Figure 1:



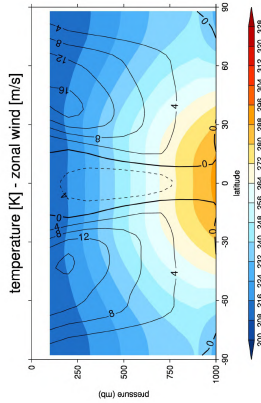
(a)  $\mathcal{F}_f = 1.5 \times 10^{-3}$ ,  $\mathcal{R}o = 8$



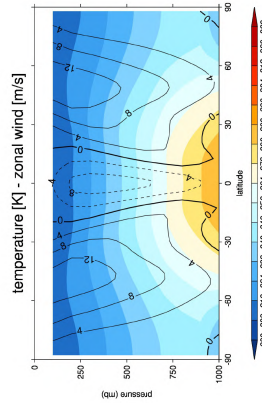
(b)  $\mathcal{F}_f = 1$ ,  $\mathcal{R}o = 8$



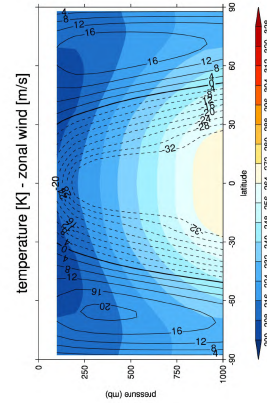
(c)  $\mathcal{F}_f = 4 \times 10^5$ ,  $\mathcal{R}o = 8$



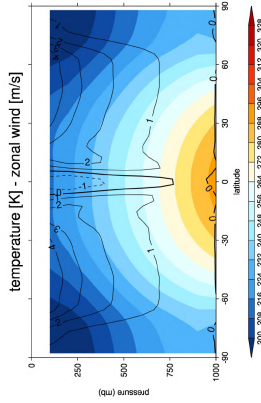
(d)  $\mathcal{F}_f = 10^{-1}$ ,  $\mathcal{R}o = 0.08$



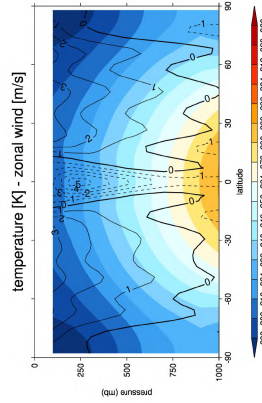
(e)  $\mathcal{F}_f = 10^2$ ,  $\mathcal{R}o = 0.08$



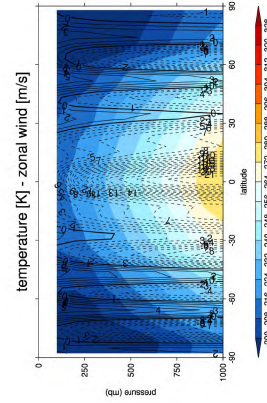
(f)  $\mathcal{F}_f = 4 \times 10^5$ ,  $\mathcal{R}o = 0.08$



(g)  $\mathcal{F}_f = 10$ ,  $\mathcal{R}o = 10^{-3}$



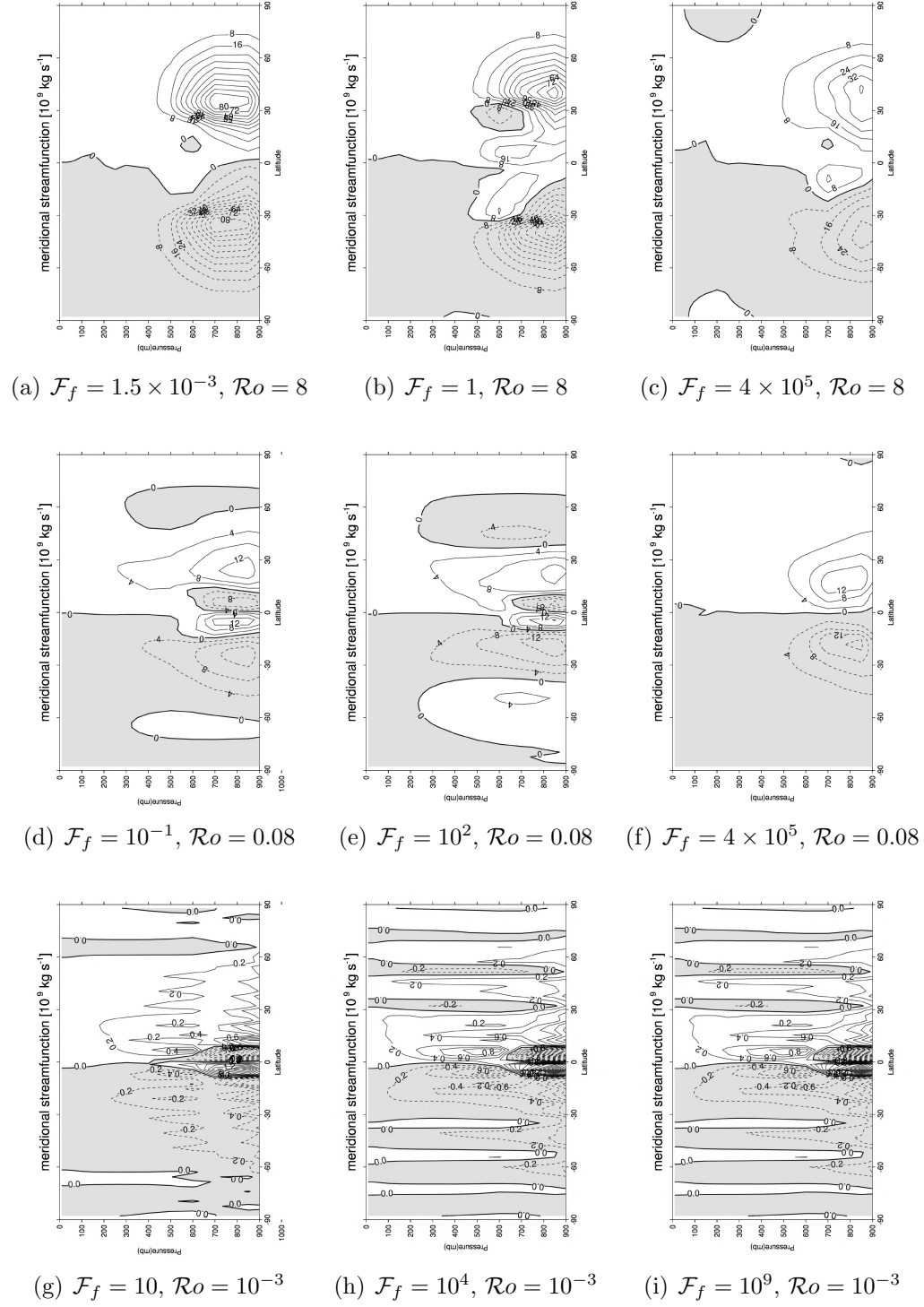
(h)  $\mathcal{F}_f = 10^4$ ,  $\mathcal{R}o = 10^{-3}$



(i)  $\mathcal{F}_f = 10^9$ ,  $\mathcal{R}o = 10^{-3}$

Figure 2:





39  
Figure 3:

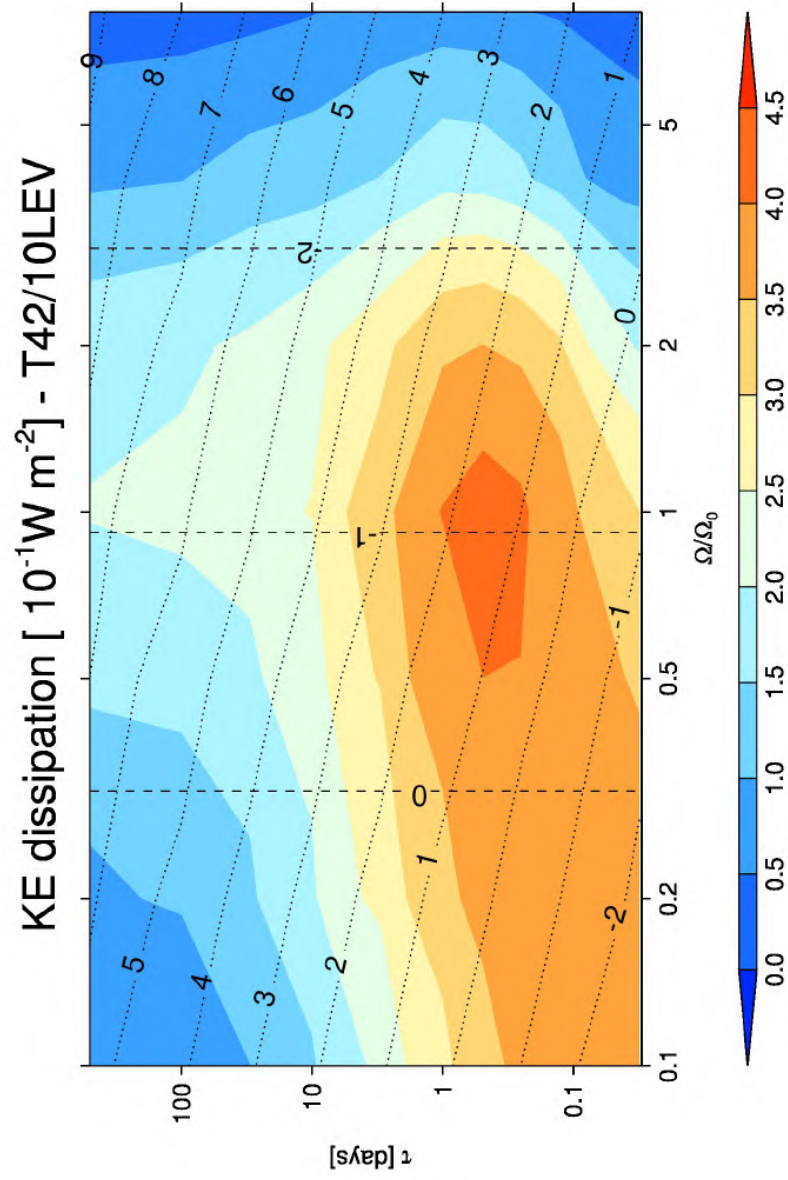


Figure 4:

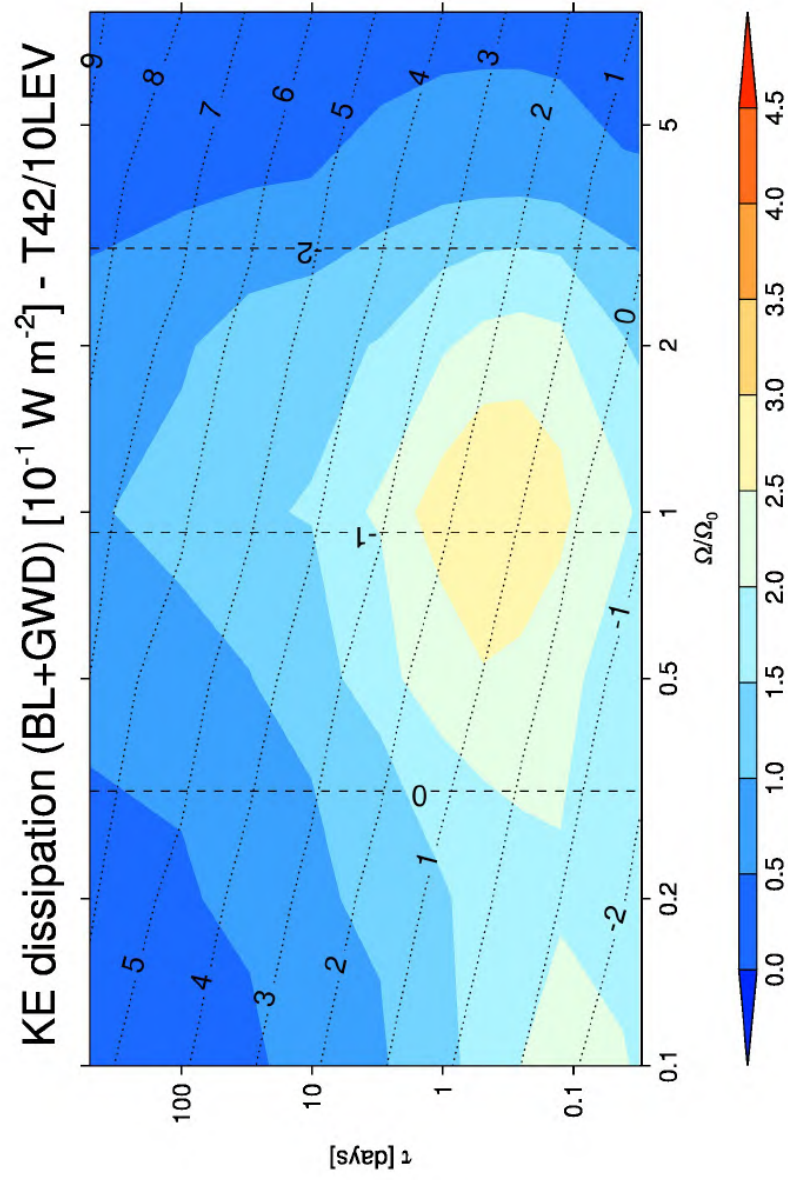


Figure 5:

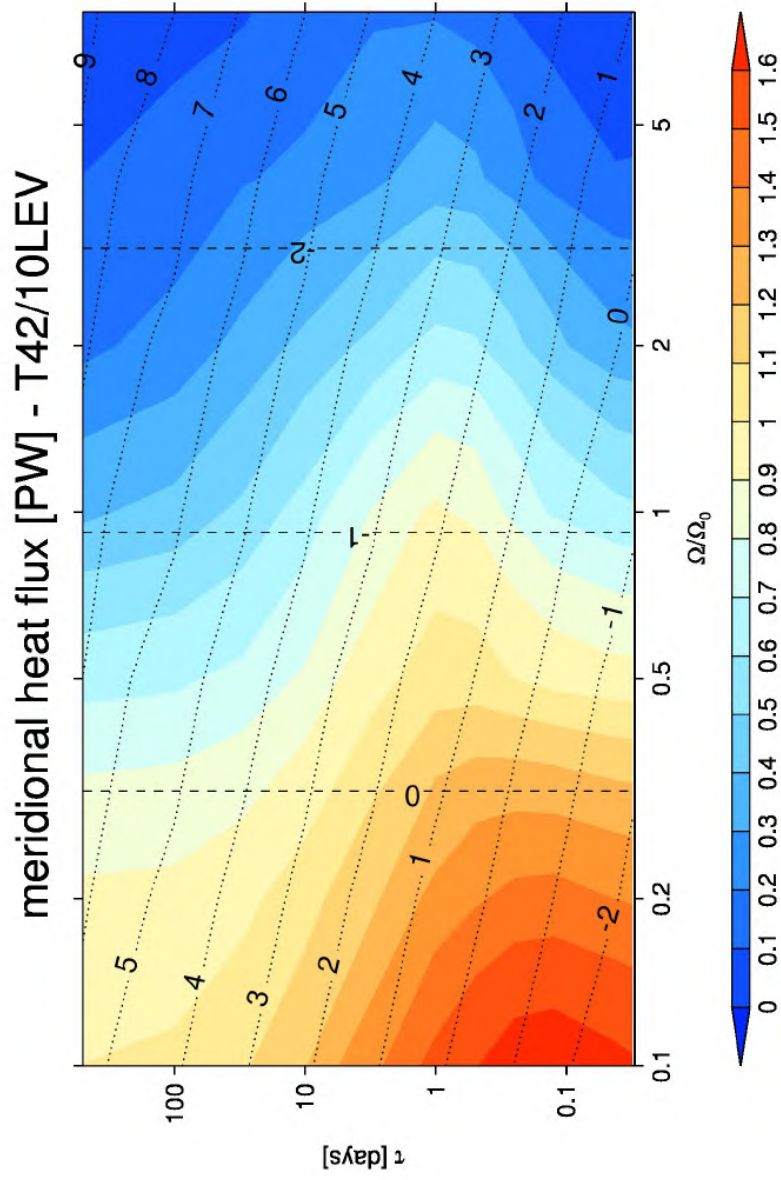


Figure 6:

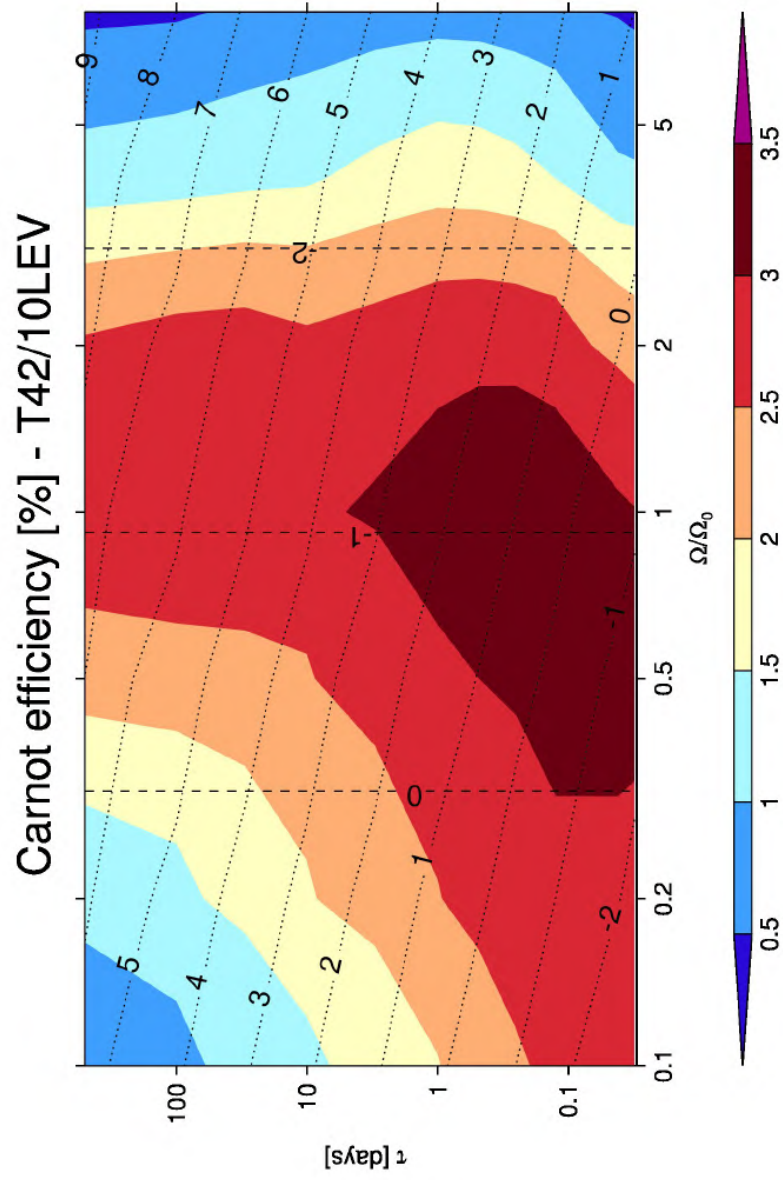


Figure 7:

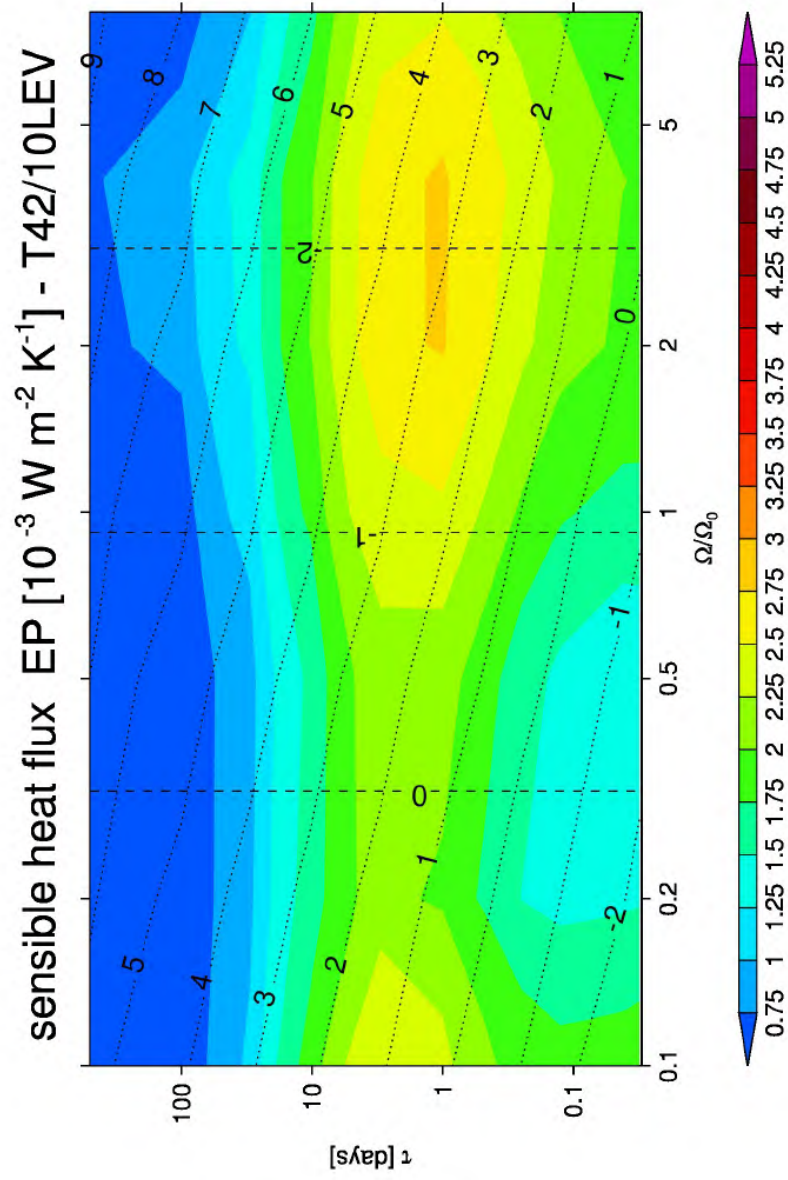


Figure 8:



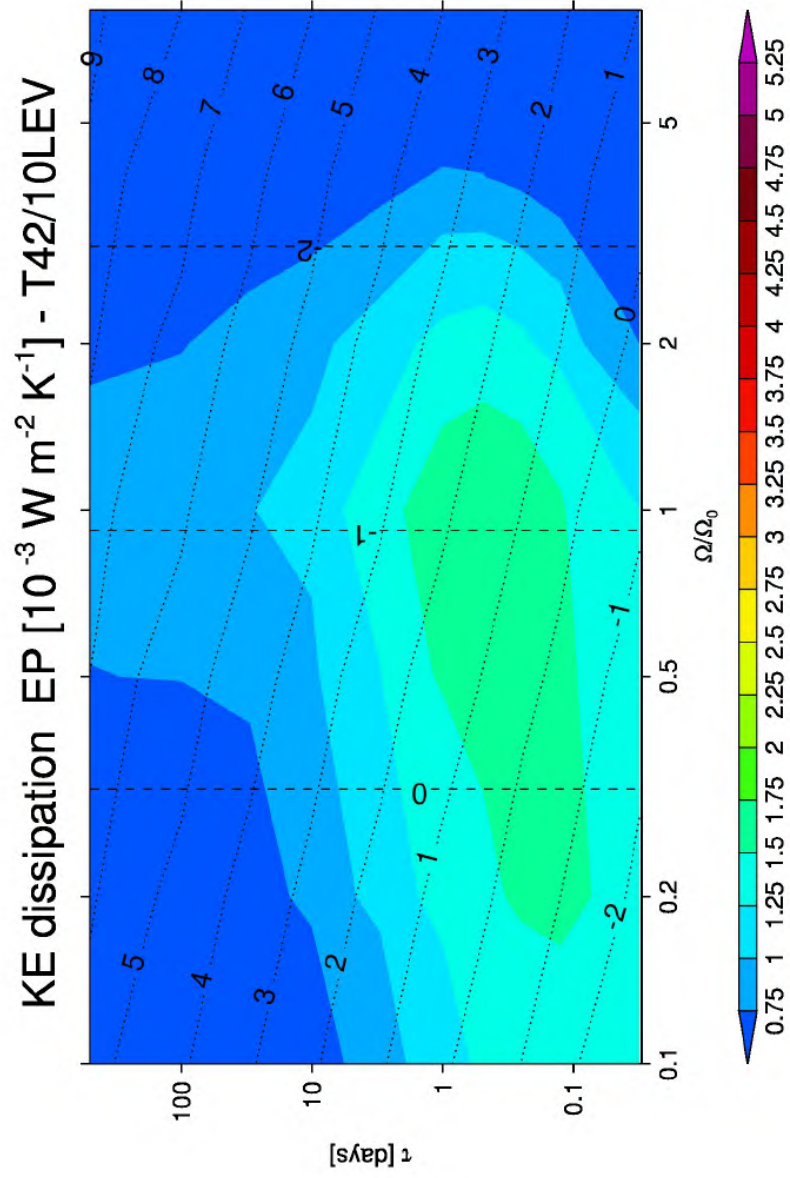


Figure 9:

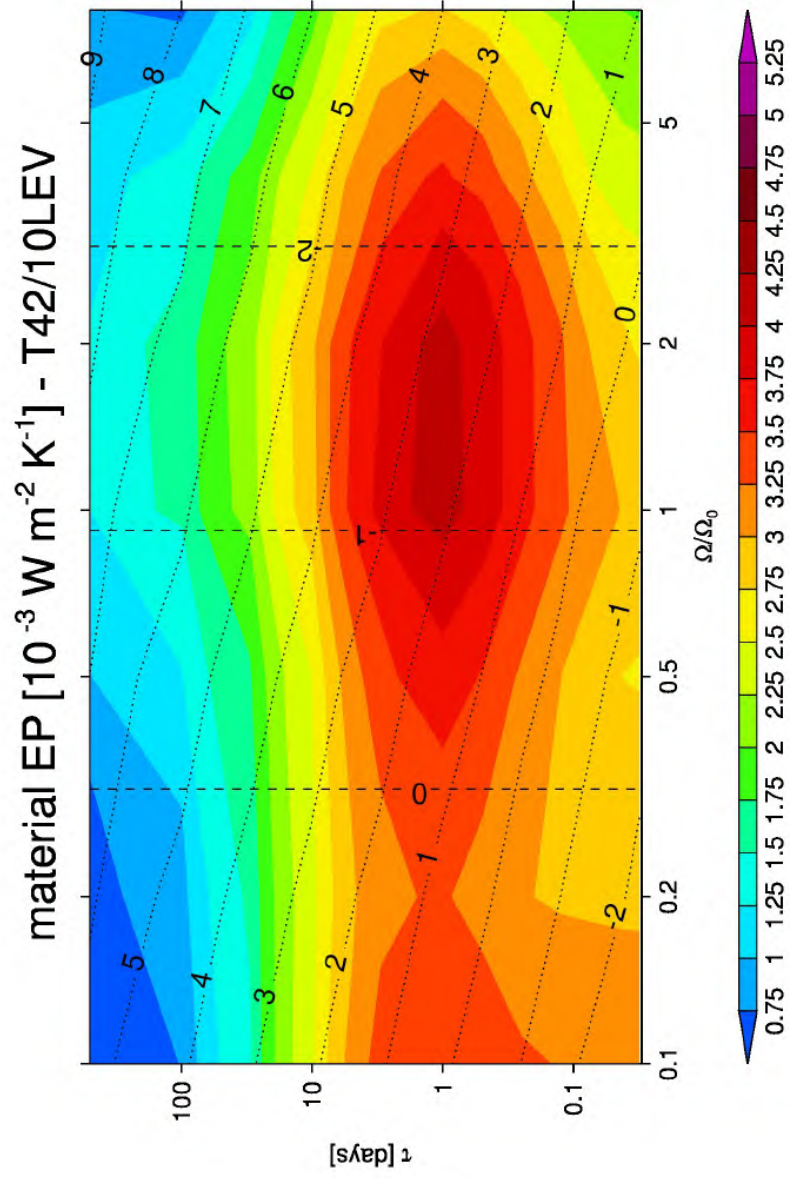


Figure 10:



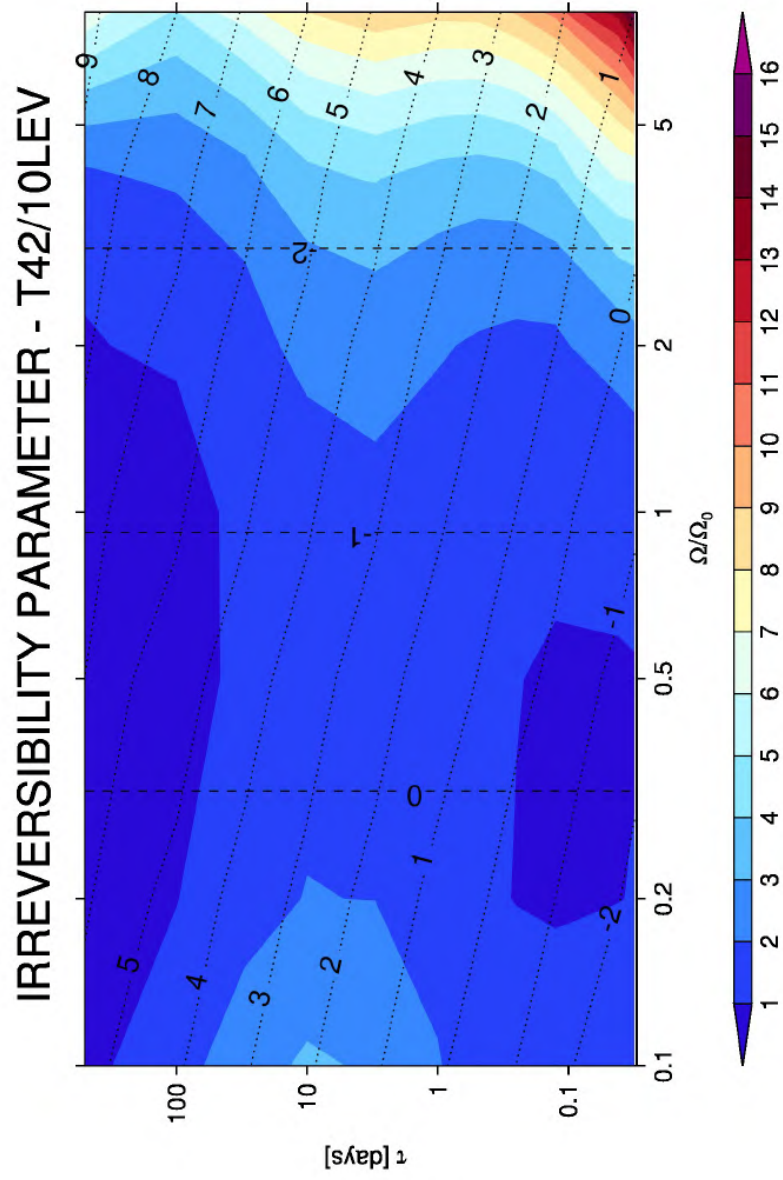
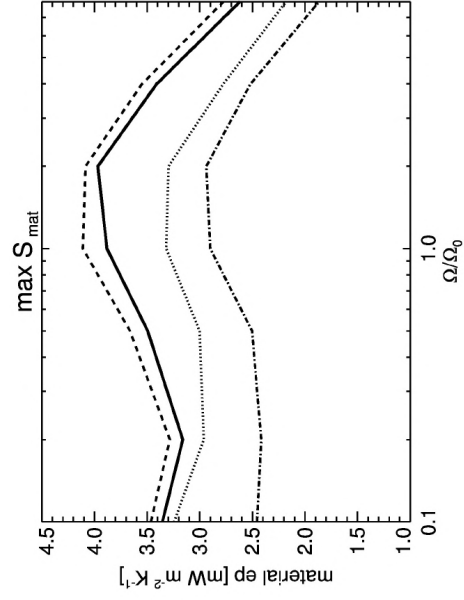
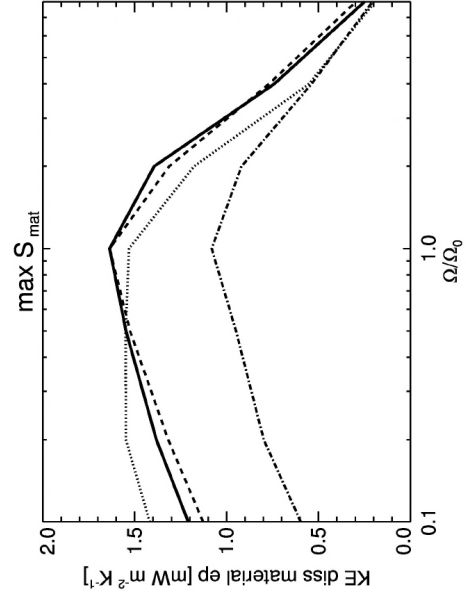


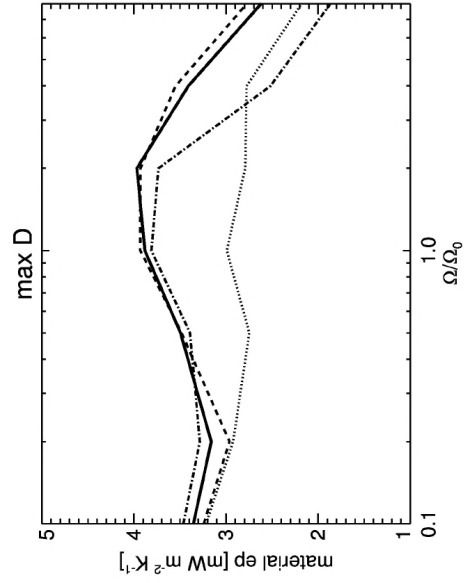
Figure 11:



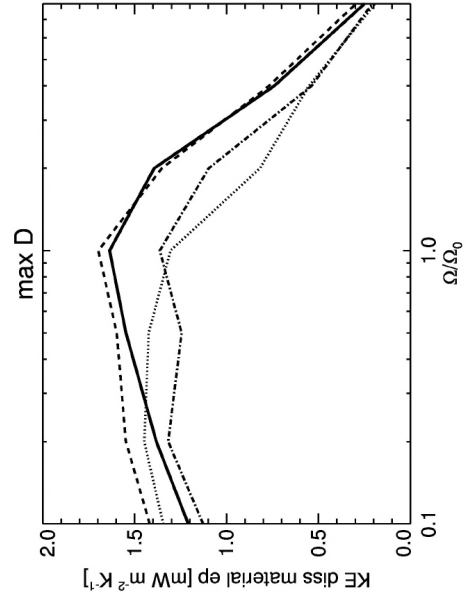
(a)



(b)



(c)



(d)

Figure 12:

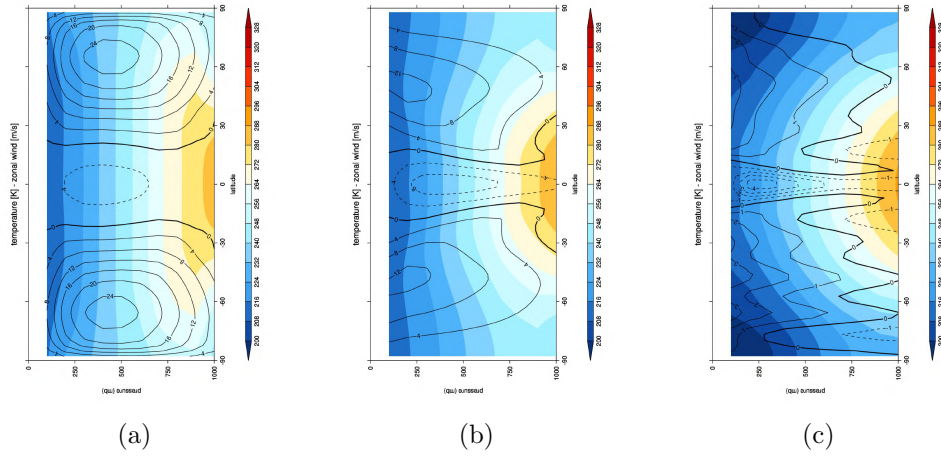


Figure 13:

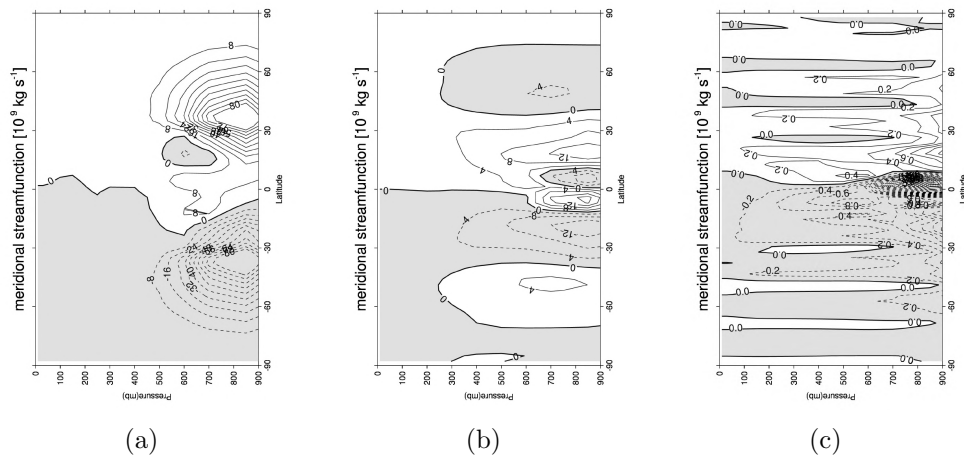


Figure 14: